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The Mathematics Content Authority List for K-6 represents an attempt to list in one volume the mathematics concepts and skills currently taught in the elementary schools. The list consists of approximately 300 items of mathematical definitions, explanations, and examples. This list is used by the Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES) project to code activities in 20 elementary school mathematics series. A school staff can determine the content and sequence they desire for a mathematics program and compare their plan with the analyses of the textbook series. By using this procedure a school staff can select a textbook or combination of textbooks to provide appropriate materials for their students. (RP)

PENNSYLVANIA DEPARTMENT OF PUBLIC INSTRUCTION

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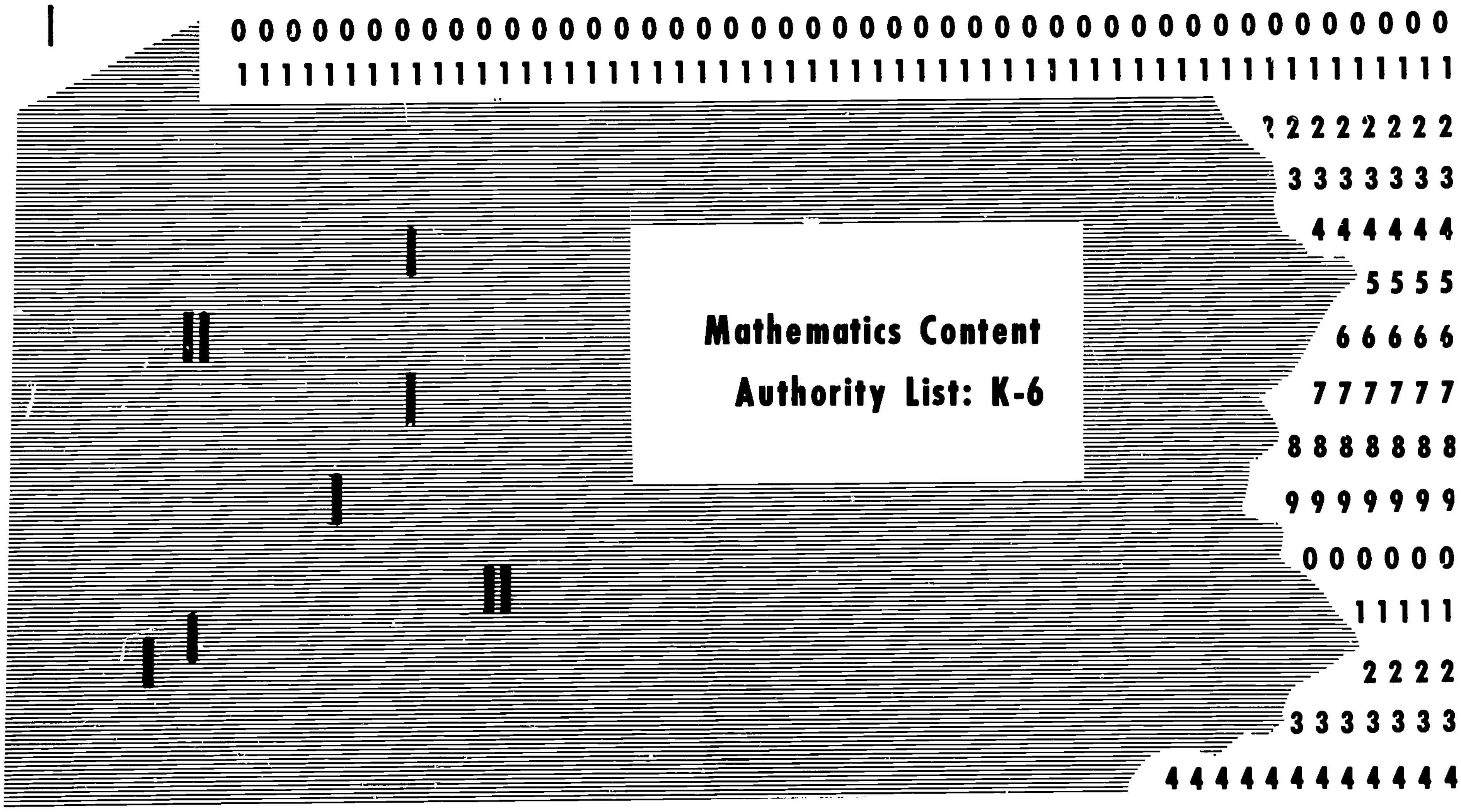

Mathematics Content Authority List: K-6

PRIMES

PENNSYLVANIA RETRIEVAL OF INFORMATION FOR MATHEMATICS EDUCATION SYSTEM

MATHEMATICS CONTENT AUTHORITY LIST: K-6

Department of Public Instruction
Bureau of Research
Bureau of General and
Academic Education

JANUARY, 1969

PREFACE

The *MATHEMATICS CONTENT AUTHORITY LIST: K-6* represents an attempt to collect in one volume the mathematics concepts and skills currently taught in the elementary schools. The list consists of approximately three hundred items with definitions, explanations, and examples, where appropriate. This list is used by the Pennsylvania Retrieval of Information for Mathematics Education System (PRIMES) project to code twenty elementary school mathematics series.

The *MATHEMATICS CONTENT AUTHORITY LIST* is the result of contributions made by many individuals over the last three years. But were it not for the dedication and commitment of *Dr. Joy Mahachek*, retired, Chairman, Mathematics Department, Indiana University of Pennsylvania, this publication would not have been possible. The "final" draft was carefully reviewed by *Dr. Lee Boyer*, retired, Professor, Mathematics Department, Millersville State College, and many of his suggestions were incorporated in the text.

Among the other people who participated in the development of the publication are: *Dr. J. Fred Weaver*, University of Wisconsin; *Dr. Earle E. Myers*, University of Pittsburgh; *Dr. C. Alan Riedesel*, The Pennsylvania State University, who served on a committee to review the main concepts of this publication and to edit an early draft of the content items.

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0010 . . . . a. Definition: set of whole numbers

Ex. $\{0,1,2,3,4,5,6,\ldots\}$

0019 . . . . b. Developing cardinal number sense

Cardinal number expresses the *manyness* *of a set; it tells* *how many* *elements are in a set.*

Ex. $N\{a,b,c,d\} = 4$ $N\{\ \} = 0$

0020 . . . . 1) Developing cardinal number zero

0030 . . . . 2) Developing cardinal numbers one through ten

0035 . . . . 3) Developing cardinal numbers beyond ten

0040 . . . . c. Developing ordinal number sense

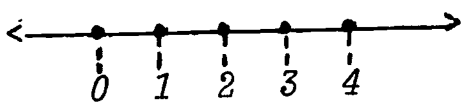
An ordinal number indicates the position of an item in a sequence of items in contrast to a cardinal number which tells how many items are in a set.

Ex. first, second, third; 4 o'clock (indicating the hour in a sequence of hours)

0050 . . . . d. Associating the idea of number with the number line (one-to-one correspondence)

A one-to-one correspondence is said to exist between two sets A and B if every member of set A can be paired with a member of set B and vice-versa.

Ex.



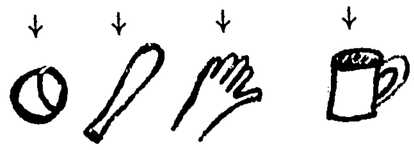
Each number on the number line corresponds to one point on the line or each number is paired with one point on the line.

0060 . . . . e. Counting to find cardinal number of set (one-to-one correspondence)

Cardinal number - See 0019, page 1

One-to-one correspondence - See 0050, page 1

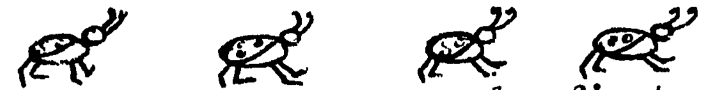
Ex. {1, 2, 3, 4 }



0070 . . . . f. Ordinal counting

Ordinal number - See 0040, page 1

Ex.



fourth third second first

0075 . . . . g. Sequence of numbers increasing by one

Ex. 1,2,3,···,
14,15,16,17,···,
31,32,33,···,

0080 . . . . h. Skip counting

Ex. 2,4,6,8,···; 5,10,15,···; 5,8,11,14,··· .

0090 . . . . i. Other counting: backward, rote, etc.

Ex. Backward: 9,8,7,6,···; 50,40,30,··· .

Rote: 1,2,3,4,5; *I caught a hare alive.*
6,7,8,9,10; *I let him go again.*

0100 . . . . j. Ordering numbers as greater than, less than, equal to or not equal to, and between, and objects as fewer than or more than

Ex. $7 > 2$; $5 < 9$; $3 + 1 = 4$; $2 \times 7 \neq 15$

7 apples are <u>more</u> than 5 apples.

A dog has <u>fewer</u> eyes than legs.

0102 . . . . . . 2. Operations

0104 . . . . a. Addition

0106 . . . . 1) Properties

0110 . . . . a) Addition, a binary operation

(1) A binary operation is an operation on two elements in a set to produce a third element belonging to the set.

(2) The binary operation of addition combines two numbers to obtain a unique result.

Ex. $2 + 3 = 5$

The number 2 and the number 3 are combined to obtain the number 5.

0120 . . . . b) Addition developed from union of disjoint sets or joining action

Disjoint sets are sets which have no elements in common.

Ex. $\{\triangle, \circ, \square\}$ *and* $\{\square, \triangle\}$ *are disjoint sets.*

$$\{\triangle, \circ, \square\} \cup \{\square, \triangle\} = \{\triangle, \circ, \square, \square, \triangle\}$$

The union of the two disjoint sets forms a new set.

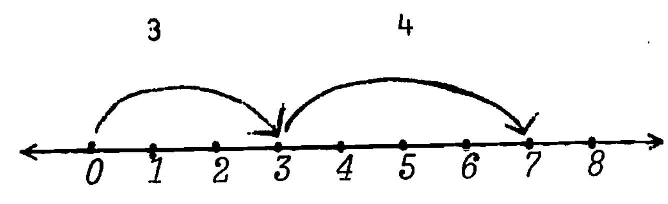
$$N\{\triangle, \circ, \square\} + N\{\square, \triangle\} = N\{\triangle, \circ, \square, \square, \triangle\}$$

Adding the numbers of the two sets gives the number of the new set.

3 + 2 = 5 (adding the cardinal numbers of the sets).

0130 . . . . c) Addition developed from number line

Ex.



$3 + 4 = 7$

Note: Some authorities prefer to use straight lines rather than curved lines.

0140 . . . . d) Closure, a property of addition

A binary operation with numbers such that the resulting number is always a member of the set being considered is said to be closed under that operation.

Since for every pair of numbers in the set of whole numbers, the unique sum is also in that set, closure is a property of addition of whole numbers.

Ex. $A = \{1,2,3,4,...\}$

$6 + 99 = 105$

6,99 and 105 are all members of set A.

Ex. $B = \{1,3,5,7,...\}$

$3 + 5 = 8$

8 is not in set B

Closure is not a property of the addition of odd numbers.

0150 . . . . e) Commutativity, a property of addition

(1) *Commutativity is another term for the commutative property.*

(2) *A binary operation is said to possess commutativity if the result of combining two elements is independent of the order.*

(3) *The commutative property is sometimes called the order property.*

Ex. The operation of addition of whole numbers possesses commutativity since

$2 + 3 = 3 + 2 = 5$

$a + b = b + a = c$

0160 . . . . f) Associativity, a property of addition

(1) *Associativity is another name for the associative property.*

(2) *An operation is said to possess associativity if the result of combining three elements by this operation is independent of the way in which the elements are combined.*

0160 (contd)

(3) The associative property is sometimes called the grouping property.

Ex. The operation of addition of whole numbers is associative.

$$5+2+1 = (5+2)+1 = 5+(2+1)$$
$$7 + 1 = 5 + 3$$
$$8 = 8$$

$$a + b + c = (a+b) + c = a + (b+c)$$

0170 g) Zero, the identity element in addition

The identity element in addition is the number which when added to any number leaves that number unchanged.

Ex. $3 + 0 = 0 + 3 = 3$

$n + 0 = 0 + n = n$

0 is the identity element for addition.

0180 h) Role of one in addition

When 1 is added to a whole number the sum is the next greater whole number.

Ex. $23 + 1 = 24$ $102 + 1 = 103$

0182 . . . . 2) Computation

0184 . . . . a) Two addends

0190 . . . . (1) Elementary facts of addition

An elementary (basic) fact of addition has two whole number addends, each less than ten.

Ex. $6 + 7 = 13$

0200 . . . . (2) Multi-digits used in addition without renaming

Renaming in addition means considering ones as ones and tens, or tens as tens and hundreds, etc.

0200 (contd)

Ex.

$$\begin{array}{r} 213 \\ +\ \underline{142} \\ 355 \end{array}$$

No renaming is necessary. See 0210 for an example using renaming (in some texts called regrouping or carrying in addition).

0210 . . . . (3) Multi-digits used in addition with renaming

Ex.

$$\begin{array}{rcl} 337 & = & 300 + 30 + 7 \\ +\ \underline{184} & = & \underline{100 + 80 + 4} \\ & & 400 + 110 + 11 \end{array}$$

$400 + (100 + 10) + (10 + 1) =$

$(400 + 100) + (10 + 10) + 1 = 521$

The 11 ones are renamed as 1 ten and 1 one. The 11 tens are renamed as 1 hundred and 1 ten. The tens are combined and the hundreds are combined giving 521.

0221 . . . . b) More than two addends

0223 . . . . (1) Single digits used in addition without renaming

Ex.

$$\begin{array}{r} 3 \\ 2 \\ +\ \underline{4} \end{array} \qquad 5 + 6 + 4 = ?$$

3+2+4 does not use renaming since neither 5+4 nor 6+3 uses renaming. 5+6+4 does not use renaming since in neither 11+4 nor 5+10 do the ones need to be renamed.

0225 . . . . (2) Single digits used in addition with renaming

When the addition fact used is greater than 9+9 renaming will be needed.

Ex.

$$\begin{array}{r} 8 \\ 9 \\ 5 \\ +\ \underline{3} \end{array}$$

Adding down 17+5 will use renaming even though addition may be done by considering the ending for the basic fact 7+5. That is, 12 will be renamed as 1 ten and 2 ones.

0227 . . . . (3) Multi-digits used in addition without renaming

Ex.

$$\begin{array}{r} 213 \\ 141 \\ +\ 234 \\ \hline 588 \end{array} \qquad \begin{array}{r} 313 \\ 241 \\ +\ 631 \\ \hline 1185 \end{array}$$

The ones do not need to be renamed as tens and ones. The tens do not need to be renamed as hundreds and tens.

0229 . . . . (4) Multi-digits used in addition with renaming

Ex.

$$\begin{array}{r} 427 \\ 356 \\ +\ 485 \\ \hline 1268 \end{array}$$

18 ones must be considered as 1 ten and 8 ones, 16 tens as 1 hundred and 6 tens, etc.

0232 . . . . b. Subtraction

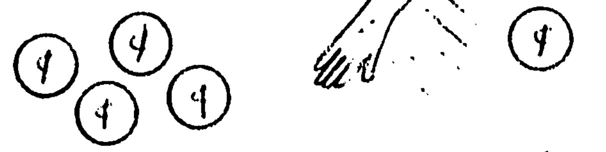
0234 . . . . 1) Properties

0240 . . . . a) Subtraction, a binary operation

Binary operation - See 0110, page 3

0250 . . . . b) Subtraction developed in relation to subsets or separating action

Ex. *Pennies*

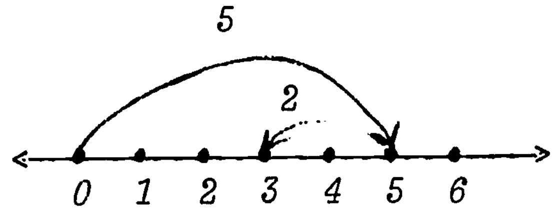


Separate one penny from the set of 5 pennies

5 - 1 = 4

0260 . . . . c) Subtraction developed from number line

Ex.



5 - 2 = 3

0270 . . . . d) Subtraction, the inverse of addition (relationship of addition and subtraction)

An inverse operation is one which undoes another operation, as standing is the inverse of sitting and sitting is the inverse of standing.

Ex. $8 + 3 = 11$ *and* $11 - 3 = 8$

Adding the number 3 to the number 8 gives the sum 11 ($8 + 3 = 11$).

Subtracting the number 3 from the sum 11 gives the missing addend 8 ($11 - 3 = 8$).

Subtraction is the inverse of addition.

0280 . . . . e) Role of zero in subtraction

Ex. $8 - 0 = 8$ $n - 0 = n$

Zero is the righthand identity element for subtraction.

$8 - 8 = 0$ $n - n = 0$

Any number subtracted from itself is zero.

0290 . . . . f) Nonclosure, noncommutativity, nonassociativity of subtraction of whole numbers

Closure - See 0140, page 4
Commutativity - See 0150, page 4
Associativity - See 0160, page 4

If $A = \{1,2,3,4,\cdots\}$ *and the operation is subtraction, then closure is not a property of the operation.*

Ex. $3 - 8 = {}^{-}5$ *but* ${}^{-}5$ *is not a member of set A.*

Commutativity is not a property of subtraction.

0290 (contd) *Ex.* $7 - 3 \neq 3 - 7$ *since* $4 \neq {}^{-}4$

Associativity is not a property of subtraction.

Ex. $(9-2)-1 \neq 9-(2-1)$ *since* $7 - 1 \neq 9 - 1$

$6 \neq 8$

0300 . . . . g) **Role of one in subtraction**

Subtracting 1 from a whole number gives the next lesser number.

Ex. $7 - 1 = 6$ $36 - 1 = 35$

0302 . . . . 2) **Computation**

0310 . . . . a) **Elementary facts of subtraction**

An elementary (basic) fact of subtraction has <u>*two*</u> *whole number addends, known and missing, each less than ten.*

Ex. $16 - 9 = 7$

$16 - \square = 7$

$16 - \triangle = 9$

$16 - 9 = \triangle$

The addends 9 and 7 are both less than 10.

0320 . . . . b) **Multi-digits used in subtraction without renaming**

Renaming <u>*in subtraction*</u> *means to consider 1 ten as* <u>*10*</u> *ones or one hundred as 10 tens, etc.*

Ex.
$$\begin{array}{r} 47 \\ -\underline{23} \\ 24 \end{array}$$
No renaming is necessary.

0330 . . . . c) **Multi-digits used in subtraction with renaming**

Ex.
$$\begin{array}{r} 52 \\ -\underline{25} \end{array}$$
5 tens and 2 ones may be renamed as 4 tens + 12 ones

then
$$\begin{array}{r} 40 + 12 \\ -\underline{(20 + \ \ 5)} \\ 20 + \ \ 7 \end{array}$$
or 27

0332 . . . . . . c. Multiplication

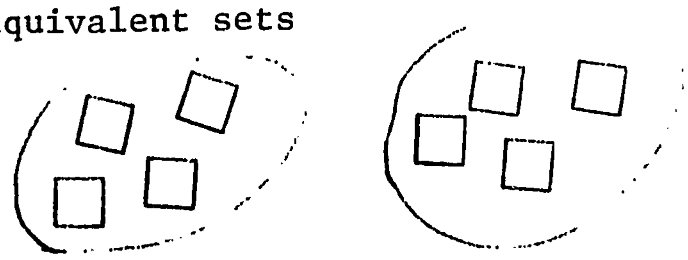
0334 . . . . 1) Properties

0340 . . . . a) Multiplication, a binary operation

Binary operation - See 0110, page 3

0350 . . . . b) Multiplication developed from union of two or more equivalent sets

Ex.

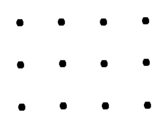


Two sets of 4 are equivalent to one set of 8
Two 4's are 8
2 x 4 = 8

0360 . . . . c) Multiplication developed from arrays

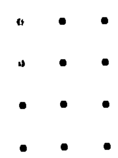
An array is an orderly arrangement of objects in rows and columns.

Ex.



3 fours are 12 or 3 x 4 = 12

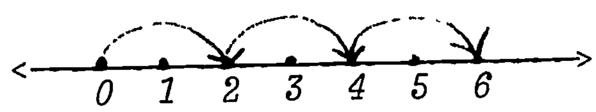
Ex.



4 threes are 12 or 4 x 3 = 12

0370 . . . . d) Multiplication developed from the number line

Ex.



Three 2's are 6
3 x 2 = 6

0380 . . . . e) Multiplication developed as repeated addition

Ex. $4 + 4 + 4 = 12$

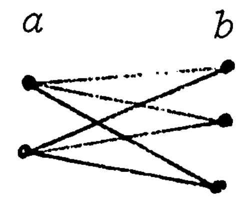
The sum of three 4's is 12

$3 \times 4 = 12$

0390 . . . . f) Multiplication developed from Cartesian product sets

Cartesian product sets - See 4160, page

Ex.



How many dots in column a? in column b? Connect each dot in a with every dot in b. How many line segments did you draw? (6)

The first dot in column a is connected with 3 dots in column b by 3 different line segments. The second dot in column a is connected to the same 3 dots in column b by 3 different line segments. Together the dots are connected by 2 x 3 or 6 lines.

0400 . . . . g) Closure, a property of multiplication

Closure - See 0140, page 4

Ex. $12 \times 25 = 300$

12, 25 and 300 are all numbers in the set of whole numbers.

0410 . . . . h) Commutativity, a property of multiplication

Commutativity - See 0150, page 4

Ex. $2 \times 3 = 3 \times 2 = 6$ $\qquad a \times b = b \times a$

The product is independent of the order of the factors.

0420 . . . . i) Associativity, a property of multiplication

Associativity - See 0160, page 4

Ex. $3 \times 2 \times 1 = (3 \times 2) \times 1 = 3 \times (2 \times 1)$

$a \times b \times c = (a \times b) \times c = a \times (b \times c)$

The product is independent of the way in which the factors are associated.

0430 . . . . j) Distributivity, a property of multiplication over addition or subtraction

(1) Distributivity is another name for the distributive property.

(2) An operation is said to possess distributivity if when performed on a set of quantities the result is equal to performing the operation on the individual members of the set and combining the results.

Multiplication is distributed over addition or subtraction.

Ex. $4x(3+2) = (4x3)+(4x2) = 12+8=20$ *and* $4x5 =$

$4x(3-1) = (4x3)-(4x1) = 12-4= 8$ *and* $4x2 =$

$a \times (b+c) = ab + ac$

0440 . . . . k) One, the identity element in multiplication

Identity element - See 0170, page 5

1 is the identity element for multiplication because multiplying a number by 1 leaves that number unchanged.

Ex. $5 \times 1 = 1 \times 5 = 5$

$1 \times 3 = 3 \times 1 = 3$

$n \times 1 = 1 \times n = n$

0450 . . . . l) Property of zero in multiplication

Any number times zero equals zero

Ex. $5 \times 0 = 0 \times 5 = 0$

$n \times 0 = 0 \times n = 0$

0452 . . . . 2) Computation

0454 . . . . a) Two factors

0460 . . . . (1) Elementary facts of multiplication

An elementary (basic) fact of multiplication has two whole number factors, each less than ten.

Ex. $5 \times 7 = 35$

0470 . . . . (2) Multi-digits used in multiplication without renaming

Renaming in multiplication means considering ones as tens and ones, tens as hundreds and tens, etc.

Ex.

$$\begin{array}{r} 32 \\ \times\ 3 \\ \hline 96 \end{array} \qquad \begin{array}{r} 42 \\ \times\ 3 \\ \hline 126 \end{array}$$

There is no need to consider ones as tens and ones nor tens as hundreds and tens.

0480 . . . . (3) Multi-digits used in multiplication with renaming

Ex.

$$\begin{array}{r} 45 \\ \times\ 7 \\ \hline 315 \end{array} \qquad \begin{array}{r} 40+5 \\ \times\ 7 \\ \hline 280+35 \end{array} =$$

$$280+(30+5) =$$

$$(280+30)+5 = 310+5 = 315$$

Note that 35 was considered as 3 tens + 5 ones. 3 tens were then added to 28 tens.

0490 . . . . b) Multiplication with more than two factors, without renaming

Ex. $2 \times 3 \times 4$

$2 \times 3 \times 9$

$1 \times 2 \times 3 \times 4$

Only elementary facts are used.

0500 . . . . c) Multiplication with more than two factors, with renaming

Ex. $6x3x9 = (6x3)x9 = 6x(3x9)$

$18x9 = 6x27$

$162 = 162$

When 8 is multiplied by 9 the 72 must be considered as 7 tens and 2 ones and 7 tens combined with 9 tens; or when 7 is multiplied by 6 the 42 must be considered as 4 tens and 2 ones and the 4 tens added to 12 tens.

0510 . . . . d) Multiples of ten as a factor

Multiples of 10 are numbers which have a factor of 10 such as 10, 20, 60, 120.

Ex.

$$\begin{array}{r} 12 \\ x\ \underline{10} \\ 120 \end{array} \qquad \begin{array}{r} 13 \\ x\ \underline{20} \\ 260 \end{array} \qquad \begin{array}{r} 25 \\ x\ \underline{60} \\ 1500 \end{array}$$

0515 . . . . e) A power of ten as a factor

Numbers such as 10^2, 10^3, 10^5 *etc., are whole number powers of 10.*

Ex. $100 \times 15 = 1{,}500 \quad (100 = 10^2)$

$1000 \times 23 = 23{,}000 \quad (1000 = 10^3)$

0520 . . . . f) A number expressed in exponential form as a factor

An exponent is a small numeral written above and to the right of a base numeral. When the exponent is a whole number it shows how many times the base is used as a factor.

Ex. $4^3 \times 5 = 4 \times 4 \times 4 \times 5$

$a^2 \times a^3 = a^{2+3} = a^5$

$6^2 \times 6^3 = 6^5$

$81 \times 3^3 = 3^4 \times 3^3 = 3^7$

$9^2 \times 3^3 = 3^2 \times 3^2 \times 3^3 = 3^7$

0522 . . . . d. Division

0524 . . . . 1) Properties

0530 . . . . a) Division, a binary operation

Binary operation - See 0110, page 3

0540 . . . . b) Division developed from partitioning into equivalent sets

Ex.

0 0
0 0
0 0

How many sets of 3 objects each can be formed from 6 objects or if six objects are separated into two equivalent sets how large will each set be?

0550 . . . . c) Division developed as successive subtraction

Ex.

$$\begin{array}{r} 12 \\ -\underline{4} \\ 8 \\ -\underline{4} \\ 4 \\ -\underline{4} \\ 0 \end{array}$$

In the total operation 4 was subtracted 3 times with no remainder. There are three 4's in 12 or 12 ÷ 4 = 3.

0555 . . . . d) Division developed from arrays

Arrays - See 0360, page 10

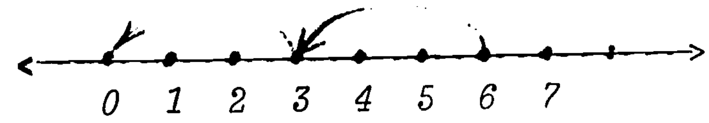
Ex. . . . .

. . . .

If an array of 8 members is arranged with 4 to a row, how many rows will there be? 8 ÷ 4 = 2; or if an array of eight members is arranged in two rows, how many items will be in each row? 8 ÷ 2 = 4.

0560 . . . . e) Division developed from the number line

Ex.



How many 3's are there in 6?

$6 \div 3 = 2$

0570 . . . . f) Division, the inverse of multiplication

Inverse - See 0270, page 8

Ex. If $4 \times \square = 12$

then $12 \div 4 = \square$ *(that same number)*

0575 . . . . g) Distributivity of division over addition or subtraction

Ex. $(4+8) \div 2 = (4 \div 2) + (8 \div 2)$
$12 \div 2 = 6$ *and* $2+4 = 6$

$(8-2) \div 2 = (8 \div 2) - (2 \div 2)$
$6 \div 2 = 3$ *and* $4 - 1 = 3$

The distributivity of division holds only when the operation of addition or subtraction precedes the divisor.

0580 . . . . h) Role of one in division

Ex. $12 \div 1 = 12$; $n \div 1 = n$
1 is the right hand identity element for division.

$12 \div 12 = 1$; $n \div n = 1$

Any number except 0 divided by itself is 1.

0590 . . . . i) Zero not a divisor

What number could equal a number divided by zero? $\frac{n}{0} = ?$ *Could* $\frac{6}{0} = 0$? *Since division is the inverse of multiplication then* 0×0 *would have to equal 6. This we know is not true. No number times 0 will = 6 and no number times 0 will = n. Therefore, division by zero is an undefined operation.*

0600 . . . . j) Nonclosure, noncommutativity, nonassociativity of division

Closure. - See 0140, page 4
Commutativity - See 0150, page 4
Associativity - See 0160, page 4

If $A = \{1,2,3,4\cdots\}$ *and the operation is division, then*

Closure is not a property of division since $3 \div 12 = 1/4$, *but 1/4 is not a member of set A.*

Commutativity is not a property of division since $12 \div 3 \neq 3 \div 12$ *as* $4 \neq 1/4$.

Associativity is not a property of division since $(24 \div 8) \div 2 \neq 24 \div (8 \div 2)$ *as*

$$3 \div 2 \neq 24 \div 4$$
$$3/2 \neq 6$$

0602 . . . . 2) Computation

0610 . . . . a) Elementary facts of division

In elementary (basic) facts of division, both the known factor (divisor) and unknown factor (quotient) are whole numbers each less than ten.

Ex. $4\overline{)32}$ with quotient 8 *or* $32 \div 4 = 8$

0620 . . . . b) Division: known factor (divisor), less than ten, product (dividend) not renamed; no remainder

The dividend is not renamed if each digital value in the dividend is a multiple of the divisor.

Ex. $2\overline{)4682}$ = 2341 ; $4\overline{)484}$ = 121 ; $4\overline{)1248}$ = 312

0630 . . . . c) Division: known factor (divisor) is less than ten and the product (dividend) is not renamed; remainder

Ex. $4\overline{)46}$ = 11 r 2 $4\overline{)33}$ = 8 r 1

0640 . . . . d) Division: known factor (divisor) less than ten, product (dividend) renamed; no remainder

Ex. $8\overline{)256}$ = 32

The dividend is renamed as 25 tens and 6 ones and then as 24 tens and 16 ones.

0650 . . . . e) Division: known factor (divisor) less than ten, product (dividend) renamed; remainder

Ex. $8\overline{)257}$ = 32 r 1

Two hundreds, five tens are renamed as 25 tens, then 24 tens and 17 ones.

0660 . . . . f) Division by ten or greater numbers

Ex. $10\overline{)30}$ = 3 $26\overline{)546}$ = 21

$296\overline{)52721}$ = 178 r 33

0665 . . . . g) **Division by multiples of ten**

Ex. $\begin{array}{r}25\\10\overline{)250}\end{array}$; $\begin{array}{r}41\\40\overline{)1640}\end{array}$; $\begin{array}{r}5\\300\overline{)1500}\end{array}$; $\begin{array}{r}62r86\\120\overline{)7526}\end{array}$

<u>Note:</u> If a text develops division as a series of subtractions the same type division exercises will be used though <u>renaming</u> as shown in 0640 will not be used.

The examples used in 0620 through 0665 might be solved as follows:

See 0620 -

```
Ex.   121
      ---
        1
       20
      100
    -----
    4/484
      400
      ---
       84
       80
      ---
        4
        4
        -
```

See 0630 -

```
Ex.  11r2
     ----
        1
       10
     ----
     4/46
       40
       --
        6
        4
       --
        2
```

See 0650 -

```
Ex.   32r1
      ----
        2
       30
    -----
    8/257
      240
      ---
       17
       16
       --
        1
```

See 0660 -

```
Ex.     21
        --
         1
        20
    ------
    26/546
       520
       ---
        26
        26
        --
```

See 0665 -

```
Ex.     25
        --
         5
        20
    ------
    10/250
       200
       ---
        50
        50
       ---
```

0667 . . . . h) Division by powers of ten

Ex. $263 \div 100 = 2 \text{ r } 63$

$4256 \div 1000 = 4 \text{ r } 256$

$5000 \div 100 = 50$

0670 . . . . i) Division with numbers expressed in exponential form

Exponents - See 0520, page 14

Ex. $10^5 \div 10^2 = 10^3$

$4^6 \div 4^2 = 4^4$

Use code 0670 only when exponents and bases are positive integers.

0672 . . . . e. Combined operations (addition, subtraction, multiplication, division)

0680 . . . . 1) Two sequential operations

Ex. $4 + 8 \div 2 = ?$

Parentheses should clarify such an example.

$4 + (8 \div 2) = 4 + 4 = 8$

$(4 + 8) \div 2 = 12 \div 2 = 6$

This code should not be used for examples such as:

a) $\begin{array}{r} 32 \\ \underline{\times\ 18} \end{array}$ *though both multiplication and addition are used.*

b) $28\overline{)5656}$ *though division, multiplication, and subtraction are used.*

0690 . . . . 2) More than two sequential operations

See explanation 0680, page 20

Ex. $(4 \times 8) - 5 + (8 \div 2) =$

$32 - 5 + 4 = 31$

0700 . . . . f. Raising to powers and finding roots

Ex. $4^3 = 4 \times 4 \times 4 = 64$

A square root of 25 is 5

Note: Only whole numbers can be used in code 0700 since this topic is part of the major topic Whole Numbers. See 0004.

The square root of 25 is not considered as $25^{\frac{1}{2}}$. *See 3120.*

If the notation not operation is being developed, code 3120.

0002 . . Topic I: Number Systems

0992 . . . B. Nonnegative Rational Numbers (fractional numbers)

0994 . . . . . . 1. Basic concepts

1000 . . . . a. Definition: set of nonnegative rationals (fractional numbers)

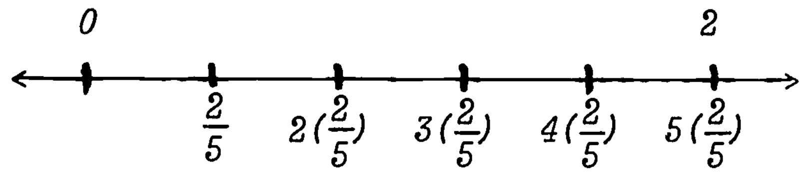
A nonnegative rational number is a number that can be expressed as the ratio of two whole numbers, provided that the second number (the divisor) is not zero.

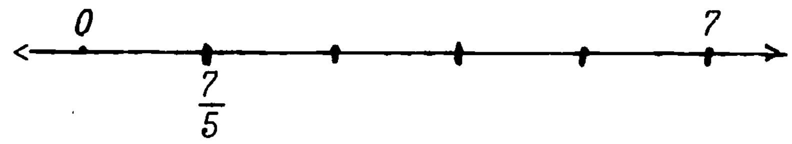
Ex. $\frac{3}{4}$; $\frac{a}{b}$ *when* $b \neq 0$

<u>Note</u>: When nonnegative rational numbers are considered, the term <u>fractional number</u> will be used. When numerals are considered, the term <u>fraction</u> will be used in the content list.

1005 . . . . b. Developed in terms of basic operations

Ex.





Name the points shown on the number line in order

$0, \frac{7}{5}, \; ? \times \frac{7}{5}, \; ? \times \frac{7}{5}, \; ? \times \frac{7}{5}, \; ? \times \frac{7}{5}$

$5 \times \frac{7}{5} = ?$

$\frac{7}{5}$ *is* $7 \div ?$

$7 \div 5$ *is* ?

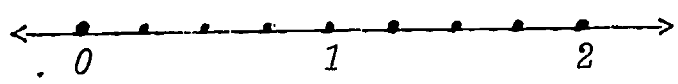
1010 . . . . c. Developed from subset of a given set

Ex. {● ○ ○ ○}

Note the set of four circles. How many are shaded? When 1 out of 4 is shaded we say 1/4 of the circles are shaded. Which numeral shows the total number of circles? Which shows the number shaded?

1020 . . . . d. Developed as distances on the number line

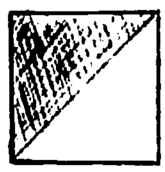
Ex.



We can pair names for fractional numbers with points on the number line. What is the name of a point half-way between 0 and 1? Let us count by halves 0/2, 1/2, 2/2, 3/2, 4/2. Divide the space from 0 to 1 into fourths and count by fourths. When we write 3/4 which numeral tells the number of equal parts into which the unit distance was divided? Which numeral tells the number of parts being considered?

1030 . . . . e. Developed from plane and solid regions

Ex.



Into how many equal parts is the square shape divided? How many parts are shaded? What part of the whole is shaded? 1 of the 2 equal parts is shaded or 1/2 of the whole is shaded. Circular shapes, candy bars, cups, etc., are often used to show fractional parts of a whole.

1035 . . . . f. Developed in other ways

1040 . . . . g. Whole numbers as related to set of nonnegative rationals (fractional numbers)

The set of nonnegative rational numbers includes the set of whole numbers which may be written in fraction form $\frac{a}{b}$; $\underline{a}$ *and* $\underline{b}$ *are whole numbers,* $\underline{a}$ *is a multiple of* $\underline{b}$ *and* $\underline{b}$ *is not zero.*

Ex. $\frac{8}{2} = 4$; $\frac{12}{4} = 3$; $\frac{10}{2} = 5$; $\frac{6}{6} = 1$

1060 . . . . h. Definition: equality

Rational numbers which have the same value are equal.

Ex. *The value of the fractional numbers* $\frac{3}{8}$ *and* $\frac{6}{16}$ *is the same.*

$\frac{a}{b} = \frac{c}{d}$ *if* $ad = bc$ *but* b *and* d *may not be zero.*

1080 . . . . i. Counting

Ex. $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}, \cdots$

$.2, .3, .4, .5, .6, \cdots$

1090 . . . . j. Ordering: greater than; less than; equal to; not equal to; between

Equality - See 1060, page 24

Ex.	$\frac{a}{b} > \frac{c}{d}$ *if* $ad > bc$	$\frac{2}{3} > \frac{7}{12}$
	$\frac{a}{b} < \frac{c}{d}$ *if* $ad < bc$	$\frac{7}{8} < \frac{11}{12}$
	$\frac{a}{b} = \frac{c}{d}$ *if* $ad = bc$	$\frac{2}{3} = \frac{4}{6}$
	$\frac{1}{2} \square \frac{1}{3} \square \frac{1}{4}$	$\frac{7}{8} \square \frac{5}{6}$

1100 . . . . k. Density

Density is a term characterizing any ordered sequence of elements such that between any two elements of the sequence another element exists. Fractional numbers have density because between any two fractional numbers another fractional number exists.

Ex. *Between* $\frac{5}{10}$ *and* $\frac{6}{10}$ *there exists another fractional number, such as* $\frac{51}{100}$*; between* $\frac{5}{10}$ *and* $\frac{51}{100}$ *there exists another fractional number, such as* $\frac{101}{200}$*; etc.*

Between .8 and .9 there exists another decimal number, .81; between .8 and .81 there exists another decimal number, .807; etc.

1102 . . . . . . 2. Operations

1104 . . . . a. Addition

1106 . . . . 1) Properties

1110 . . . . a) Addition, a binary operation

Binary operation - See 0110, page 3

Ex. $\frac{1}{2} + \frac{3}{4} = ?$

$.45 + 2.13 = ?$

1120 . . . . b) Addition developed from union of disjoint sets

Ex.

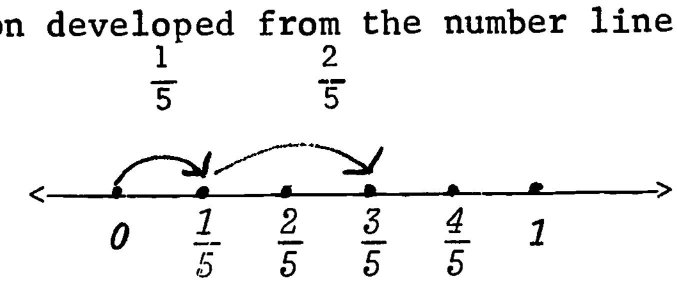
What part of the set of all the circle shapes is black? shaded? What part is colored in some way? Write a number sentence to show it.

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}.$

For decimal fractions use 10 shapes.

1130 . . . . c) Addition developed from the number line

Ex.

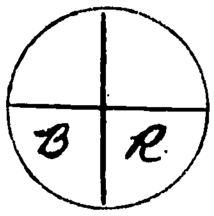


We can add fractional numbers with the same denominators as we added whole numbers $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

How shall we show the jumps? What is the denominator of the sum? How can you find the numerator of the sum?

1140 . . . . d) Addition developed from plane or solid regions

Ex.



What part of the circular shape is red? blue? What part is colored?

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$. *What does the 4 tell us? the 2? How could 2 be found from the numerators? Why must the 4 be used for all denominators? (It tells into how many equal parts the whole is separated).*

1150 . . . . e) Closure, a property of addition

Closure - See 0140, page 3

Ex. $\frac{2}{3} + \frac{3}{8} = \square$

$$\frac{16}{24} + \frac{9}{24} = \frac{25}{24}$$

$$.3 + .5 = .8$$

1160 . . . . f) Commutativity, a property of addition

Commutativity - See 0150, page 4

Ex. $\frac{2}{3} + \frac{1}{4} = \frac{1}{4} + \frac{2}{3} = \frac{11}{12}$

$$.3 + .5 = .5 + .3 = .8$$

1170 . . . . g) Associativity, a property of addition

Associativity - See 0160, page 4

Ex. $\left(\frac{1}{4} + \frac{1}{3}\right) + \frac{1}{2} = \frac{1}{4} + \left(\frac{1}{3} + \frac{1}{2}\right)$

$$\frac{7}{12} + \frac{1}{2} = \frac{1}{4} + \frac{5}{6}$$

$$\frac{7}{12} + \frac{6}{12} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12}$$

$$.3 + (.25+.4) = (.3+.25) + .4$$

$$.3 + .65 = .55 + .4 = .95$$

1180 . . . . h) Zero, the identity element in addition

Identity element - See 0170, page 5

Ex. $\frac{2}{3} + 0 = 0 + \frac{2}{3} = \frac{2}{3}$

$.4 + 0 = 0 + .4 = .4$

1182 . . . . 2) Computation

1190 . . . . a) Addition with common fraction notation, equal denominators (like fractions)

Ex. $\frac{3}{8} + \frac{1}{8} = \frac{4}{8} \text{ or } \frac{1}{2}$

$$\begin{array}{r} \frac{3}{8} \\ + \frac{1}{8} \\ \hline \frac{4}{8} \text{ or } \frac{1}{2} \end{array}$$

$$\begin{array}{r} 3 \\ + \frac{1}{4} \\ \hline 3\frac{1}{4} \end{array} \qquad \begin{array}{r} 5\frac{1}{7} \\ + 2\frac{3}{7} \\ \hline 7\frac{4}{7} \end{array} \qquad \begin{array}{r} 6 \\ 98\frac{1}{8} \\ 127\frac{3}{8} \\ + \frac{5}{8} \\ \hline 231\frac{9}{8} \text{ or } 232\frac{1}{8} \end{array}$$

1200 . . . . b) Addition with common fraction notation, unequal denominators (unlike fractions)

Ex. $\frac{2}{3} + \frac{1}{2} = \square$ or $\begin{array}{r} \frac{2}{3} = \frac{4}{6} \\ + \frac{1}{2} = \frac{3}{6} \\ \hline \frac{7}{6} \text{ or } 1\frac{1}{6} \end{array}$

$\frac{4}{6} + \frac{3}{6} = \frac{7}{6} \text{ or } 1\frac{1}{6}$

$$\begin{array}{r} 12\frac{7}{8} \\ + 6\frac{3}{4} \\ \hline 19\frac{5}{8} \end{array} \qquad \begin{array}{r} \frac{3}{8} \\ \frac{2}{3} \\ + \frac{1}{6} \\ \hline 1\frac{5}{24} \end{array}$$

1210 . . . . c) Addition with exact decimal fraction notation

Ex. $.5 + .25 = .75$ or $\begin{array}{r} .50 \\ +\ .25 \\ \hline .75 \end{array}$ (since $.5 = .50$)

$\begin{array}{r} .121 \\ .342 \\ +\ .514 \\ \hline .977 \end{array}$ $\qquad$ $\begin{array}{r} 30.7 \\ 485.2 \\ +\ 1.96 \\ \hline 517.86 \end{array}$

Do <u>not</u> use this code with addition involving money. Use 6040 and appropriate code under addition of whole numbers.

1212 . . . . b. Subtraction

1214 . . . . 1) Properties

1220 . . . . a) Subtraction, a binary operation

Binary operation - See 0110, page 3

1230 . . . . b) Subtraction developed in relation to subsets

Ex.

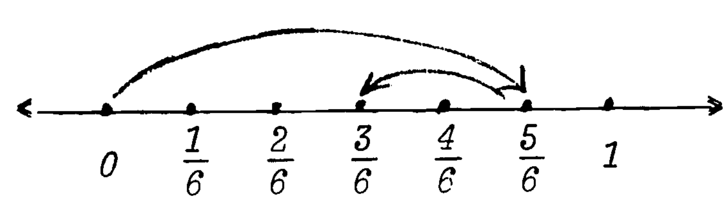
What part of the set of all the circular shapes is 1 circular shape? 2? 3? 4? 5? What part of the shapes is shaded? Can we find the part not shaded by subtracting? How?
$\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$. *Check your answer by counting.*

$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ *not shaded. For decimal fractions use*

10 shapes $\frac{10}{10} - \frac{4}{10} = \frac{6}{10}$ *or* $1 - .4 = .6$

1240 . . . . c) Subtraction developed from the number line

Ex.



1240 (contd)

Count the parts shown on the number line. Subtract $\frac{2}{6}$ from $\frac{5}{6}$ as we did with whole numbers. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$. How is the numerator found? Why is the denominator 6? (It tells into how many equal parts one unit of distance was divided in each case). For decimal fractions divide the unit distance into 10 parts.

1250 . . . . d) Subtraction developed from plane or solid regions

Ex. *See 1140, page 26*

1260 . . . . e) Subtraction, the inverse of addition

Inverse - *See 0270, page 8*

Ex. If $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ then $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$

If $\square + \frac{1}{4} = \frac{5}{4}$ then $\frac{5}{4} - \frac{1}{4} = \square$

If $\square + .31 = .56$ then $.56 - .31 = \square$

1270 . . . . f) Role of zero in subtraction

Ex. $\frac{3}{4} - 0 = \frac{3}{4}$

$\frac{a}{b} - 0 = \frac{a}{b}$

$.18 - 0 = .18$

Zero is the right hand identity element for subtraction.

$\frac{2}{3} - \frac{2}{3} = 0$

$.26 - .26 = 0$

Any number subtracted from itself is zero.

1280 . . . . g) Nonclosure, noncommutativity, nonassociativity in subtraction

Closure - See 0140, page 4
Commutativity - See 0150, page 4
Associativity - See 0160, page 4

Ex. *Nonclosure:*

$\frac{1}{2} - \frac{3}{4} = {}^{-}\frac{1}{4}$

$.5 - .75 = {}^{-}.25$

${}^{-}\frac{1}{4}$ *and* ${}^{-}.25$ *are not members of the set of nonnegative rational numbers.*

Noncommutativity:

$\frac{1}{2} - \frac{3}{4} \neq \frac{3}{4} - \frac{1}{2}$

since ${}^{-}\frac{1}{4} \neq \frac{1}{4}$

$.25 - .13 \neq .13 - .25$

Nonassociativity:

$\left(\frac{1}{2} - \frac{1}{3}\right) - \frac{1}{6} \neq \frac{1}{2} - \left(\frac{1}{3} - \frac{1}{6}\right)$

since

$\frac{1}{6} - \frac{1}{6} \neq \frac{1}{2} - \frac{1}{6}$

$0 \neq \frac{2}{6}$

$(.5 - .25) - .2 \neq .5 - (.25 - .2)$

$.25 - .2 \neq .5 - .05$

$.05 \neq .45$

1282 . . . . 2) Computation

1290 . . . . a) Subtraction with common fraction notation, equal denominators (like fractions)

Ex. $\frac{5}{6} - \frac{1}{6} = \frac{4}{6}$ *or* $\frac{2}{3}$

$$\begin{array}{r} \frac{5}{6} \\ -\ \frac{1}{6} \\ \hline \frac{4}{6} \text{ or } \frac{2}{3} \end{array}$$

1290 (contd)

$$\begin{array}{r} 3 \\ -\ \frac{1}{2} \\ \hline 2\frac{1}{2} \end{array} \qquad \begin{array}{r} 12\frac{3}{4} \\ -\ 6\frac{1}{4} \\ \hline 6\frac{2}{4} \text{ or } 6\frac{1}{2} \end{array}$$

1300 . . . . b) Subtraction with common fraction notation, unequal denominators (unlike fractions)

Ex. $\frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{3}{6} = \frac{2}{6}$ *or* $\frac{1}{3}$ *or* $\begin{array}{r} \frac{5}{6} = \frac{5}{6} \\ -\ \frac{1}{2} = \frac{3}{6} \\ \hline \frac{2}{6} \text{ or } \frac{1}{3} \end{array}$

$$\begin{array}{r} 6\frac{1}{2} \\ -\ \frac{2}{3} \\ \hline 5\frac{5}{6} \end{array} \qquad \begin{array}{r} 120\frac{11}{12} \\ -\ 17\frac{1}{3} \\ \hline 103\frac{7}{12} \end{array}$$

1310 . . . . c) Subtraction with decimal fraction notation

Ex. $\begin{array}{r} .75 \\ -\ .375 \\ \hline .375 \end{array} \qquad \begin{array}{r} .584 \\ -\ .123 \\ \hline .461 \end{array}$

Do <u>not</u> use this code with subtraction involving money. Use 6040 and appropriate code under subtraction of whole numbers.

1312 . . . . c. Multiplication

1314 . . . . 1) Properties

1320 . . . . a) Multiplication, a binary operation

Binary operation - See 0110, page 3

Ex. $\frac{2}{3} \times \frac{6}{7} = \square$

$.3 \times .4 = \square$

1330 . . . . b) Multiplication developed from addition of two or more equal fractions

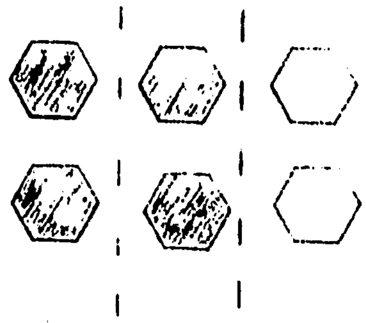
Ex. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ $\qquad .3 + .3 = .6$

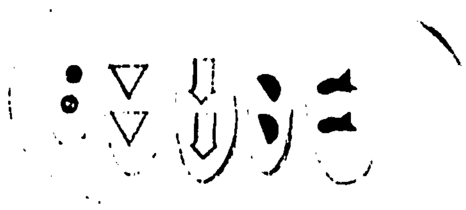
$2 \times \frac{1}{3} = \frac{2}{3}$ $\qquad 2 \times .3 = .6$

1340 . . . . c) Multiplication developed from arrays or sets

Arrays - See 0360, page 10

Ex.



A = 

$\frac{3}{5} \times 10 = 6$

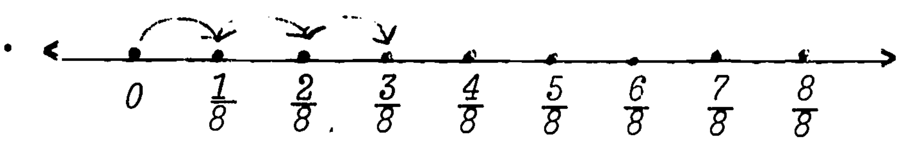
$\frac{2}{3} \times 6 = 4$

6 of the objects in set A are shaded.

$\frac{3}{5}$ of the objects in set A are shaded.

Note: One of the factors will be a whole number.

1345 . . . . d) Multiplication developed from the number line

Ex. 

To find $3 \times \frac{1}{8}$ take 3 jumps of $\frac{1}{8}$ each on the number line $3 \times \frac{1}{8} = \frac{3}{8}$. Find $3 \times \frac{2}{8} = \frac{6}{8}$. How is the new numerator found? Find also $\frac{1}{2}$ of $\frac{5}{8}$. Where is the half way point from 0 to $\frac{5}{8}$? $\left(\frac{5}{16}\right)$ Then $\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$. How is the new numerator found? the new denominator?

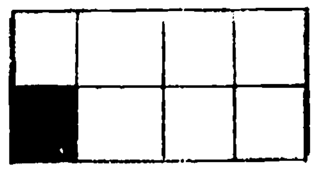
1345 (contd)

For decimal fractions number the points as .1, .2, .3, etc. 2 jumps of .3 each will bring the arrow to .6.

$$2 \times .3 = .6$$

1350 . . . . e) Multiplication developed from plane and solid regions

Ex.



The shaded part is what part of the lower row? $\left(\frac{1}{4}\right)$; *of the first column?* $\left(\frac{1}{2}\right)$; *of the whole figure?* $\left(\frac{1}{8}\right)$. *Is* $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ *a true statement? How is the new denominator found? the new numerator?*

In decimal fractions is $.5 \times .25 = .125$ *a true statement? Is one square .125 of the whole?*

1360 . . . . f) Closure, a property of multiplication

Closure - See 0140, page 4

1370 . . . . g) Commutativity, a property of multiplication

Commutativity - See 0150, page 4

Ex. $\frac{2}{3} \times \frac{4}{9} = \frac{4}{9} \times \frac{2}{3} = \frac{8}{27}$

$$.5 \times .3 = .3 \times .5 = .15$$

1380 . . . . h) Associativity, a property of multiplication

Associativity - See 0160, page 4

Ex. $\left(\frac{2}{3} \times \frac{1}{4}\right) \times \frac{1}{8} = \frac{2}{3} \times \left(\frac{1}{4} \times \frac{1}{8}\right)$

$$\frac{1}{6} \times \frac{1}{8} = \frac{2}{3} \times \frac{1}{32}$$

$$\frac{1}{48} = \frac{1}{48}$$

1380 (contd)

$.3 \times (.4 \times .5) = (.3 \times .4) \times .5$

$.3 \times .20 = .12 \times .5 = .060$

1390 . . . . i) Distributivity, a property of multiplication over addition or subtraction

Distributivity - See 0430, page 10

Ex. $\frac{2}{3}\left(\frac{1}{2} + \frac{1}{4}\right) = \left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right)$

$\frac{2}{3} \times \frac{3}{4} = \frac{1}{3} + \frac{1}{6}$

$\frac{1}{2} = \frac{1}{2}$

$\frac{2}{3}\left(\frac{1}{2} - \frac{1}{4}\right) = \left(\frac{2}{3} \times \frac{1}{2}\right) - \left(\frac{2}{3} \times \frac{1}{4}\right)$

$\frac{2}{3} \times \frac{1}{4} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

$.3 \times (.4 + .5) = (.3 \times .4) + (.3 \times .5)$

$.3 \times .9 = .12 + .15$

$.27 = .27$

1400 . . . . j) One, the identity element in multiplication

Identity - See 0170, page 5

Ex. $1 \times \frac{3}{8} = \frac{3}{8} \times 1 = \frac{3}{8}$

$1 \times .4 = .4 \times 1 = .4$

1410 . . . . k) Role of zero in multiplication

Ex. $\frac{a}{b} \times 0 = 0 \times \frac{a}{b}$ *where* $b \neq 0$

Any number times zero is zero.

$\frac{2}{3} \times 0 = 0 \times \frac{2}{3} = 0$

$.3 \times 0 = 0 \times .3 = 0$

1420 . . . . *l*) Multiplicative inverse (reciprocal) for any fractional number greater than zero

(1) *If the product of two numbers is 1, then each number is the multiplicative inverse of the other.*

(2) *The reciprocal of a number is its multiplicative inverse.*

Ex. $3 \times \frac{1}{3} = 1.$ *3 is the multiplicative inverse and the reciprocal of* $\frac{1}{3}$.

$\frac{1}{3}$ *is the multiplicative inverse and the reciprocal of 3.*

1422 . . . . 2) Computation

1430 . . . . a) Multiplication with common fraction notation

Ex. $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$ *or* $\frac{1}{6}$

$6 \times \frac{2}{3} = \frac{12}{3}$ *or* 4 ; $\frac{2}{3}$ *of* $6 = \frac{12}{3}$ *or* 4

$7\frac{1}{2} \times \frac{5}{6} = \frac{15}{2} \times \frac{5}{6} = \frac{75}{12}$ *or* $6\frac{1}{4}$

$\frac{5}{6} \times (7+\frac{1}{2}) = \frac{5}{6} \times 7 + \frac{5}{6} \times \frac{1}{2} = \frac{35}{6} + \frac{5}{12} = \frac{75}{12}$ *or* $6\frac{1}{4}$

1440 . . . . b) Multiplication with decimal fraction notation

Ex. $3 \times .18 = .54$

$.8 \times .3 = .24$

$.5 \times .25 = .125$ *or* $\begin{array}{r} .25 \\ \times \underline{\ .5} \\ .125 \end{array}$

Do *<u>not</u>* *use this code with multiplication involving money. Use 6040 and appropriate code under multiplication of whole numbers.*

1441 . . . . c) Multiplication by powers or multiples of ten

Ex. $10^2 \times \frac{7}{100} = 10^2 \times .07 = 7$ *(10^2 is written as a power of ten)*

$30 \times \frac{7}{10} = 21$ *(30 and 300 are multiples of 10)*

$300 \times \frac{7}{100} = 21$

1442 . . . . d. Division

1444 . . . . 1) Properties

1450 . . . . a) Division, a binary operation

Binary - See 0110, page 3

1460 . . . . b) Division developed from successive subtraction of two or more equal fractional numbers

Ex. $\frac{3}{4} \div \frac{1}{4} = \square$

$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

$\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$

$\frac{1}{4} - \frac{1}{4} = 0$

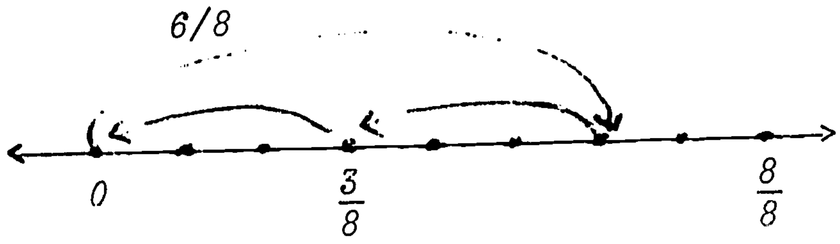
Three 1/4ths can be subtracted from 3/4 with no remainder, $\frac{3}{4} \div \frac{1}{4} = 3$.

$$
\begin{array}{rl}
.2/ & \overline{.8} \\
 & \underline{.2} \\
 & .6 \\
- & \underline{.2} \\
 & .4 \\
- & \underline{.2} \\
 & .2 \\
- & \underline{.2} \\
 & 0
\end{array}
$$

$$
\begin{array}{rl}
 & 4 \\
.2/ & \overline{.8}
\end{array}
$$

1470 . . . . c) Division developed from the number line

Ex.



$\frac{6}{8} \div \frac{3}{8} = \square$ *How many 3/8 are there in 6/8?*

What is $6 \div 3$? $8 \div 8$?

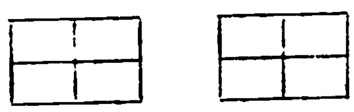
Does $\frac{2}{1}$ *name the same number as 2?*

$\frac{5}{8} \div \frac{2}{16} = \frac{10}{16} \div \frac{2}{16} = \frac{5}{1}$ *or* 5.

This may lead to $\frac{5}{8} \times \frac{16}{2} = \frac{5}{1}$ *so that the problem* $\frac{5}{8} \div \frac{2}{16}$ *may be solved by finding* $\frac{5}{8} \times \frac{16}{2}$.

1480 . . . . d) Division developed from plane and solid regions

Ex.



How many fourths are there in 2 whole figures? $2 \div \frac{1}{4} = 8$?

1490 . . . . e) Division, the inverse of multiplication with fractional numbers

Ex. $\frac{3}{8} \div \frac{1}{2} = \frac{6}{8}$

$\frac{3}{8} \times ? = \frac{6}{8}$

How is 2 related to 1/2? Write a multiplication sentence for $5 \div \frac{1}{3}$. *Does the multiplication sentence solve the division sentence?*

1500 . . . . f) Closure, a property of division

Closure - See 0140, page 4

1510 . . . . g) Role of one in division

Ex. $\frac{a}{b} \div 1 = \frac{a}{b}$ *One is right hand identity element for division*

$.8 \div 1 = .8$

$\frac{a}{b} \div \frac{a}{b} = 1$ *Any number divided by itself is one.*

$.6 \div .6 = 1$

1520 . . . . h) Zero not a divisor

Could 2/3 be divided by 0? No. There is no number times 0 which will equal 2/3.

See 0590, page 17

1530 . . . . i) Noncommutativity, nonassociativity of division

Commutativity - See 0150, page 4
Associativity - See 0160, page 4

Ex. *Noncommutativity* $\frac{2}{9} \div \frac{1}{3} \neq \frac{1}{3} \div \frac{2}{9}$

$\frac{2}{9} x \frac{3}{1} \neq \frac{1}{3} x \frac{9}{2}$

since $\frac{2}{3} \neq \frac{3}{2}$

Nonassociativity $\left(\frac{2}{9} \div \frac{1}{3}\right) \div \frac{1}{2} \neq \frac{2}{9} \div \left(\frac{1}{3} \div \frac{1}{2}\right)$

$\left(\frac{2}{9} x \frac{3}{1}\right) \div \frac{1}{2} \neq \frac{2}{9} \div \left(\frac{1}{3} x \frac{2}{1}\right)$

$\frac{2}{3} \div \frac{1}{2} \neq \frac{2}{9} \div \frac{2}{3}$

$\frac{2}{3} x \frac{2}{1} \neq \frac{2}{9} x \frac{3}{2}$

$\frac{4}{3} \neq \frac{1}{3}$

1532 . . . . 2) Computation

1540 . . . . a) Division with common fraction notation

Ex. $\frac{2}{9} \div \frac{1}{3} = \square$ *if* $\square \times \frac{1}{3} = \frac{2}{9}$

$$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

therefore $\frac{2}{9} \div \frac{1}{3} = \frac{2}{3}$

and $\frac{2}{\cancel{9}_3} \times \frac{\cancel{3}^1}{1} = \frac{2}{3}$

therefore $\frac{2}{9} \div \frac{1}{3} = \frac{2}{9} \times \frac{3}{1} = \frac{2}{3}$

1550 . . . . b) Division with decimal fraction notation

Ex. $.5 \div .25 = \square$ *if* $\square \times .25 = .5$

$.25\overline{).50}$ = 2. $\qquad 2 \times .25 = .5$

therefore $.5 \div .25 = 2$

Check $2 \times .25 = .50$

Ex. $.35\overline{)10.535}$ = 30.1

Check $.35 \times 30.1 = 10.535$

Do not use this code with division involving money. Use 6040 and appropriate code used under division of whole numbers.

1555 . . . . c) Division by powers or multiples of ten

Ex. $.563 \div 10^2 = .563 \div 100 = .00563$

$$\frac{1}{2} \div 10^3 = \frac{1}{2} \div 1000$$

$$\frac{1}{2} \times \frac{1}{1000} = \frac{1}{2000}$$

1560 . . . . e. Sequential operations

Ex. $\frac{3}{4} x \left(\frac{2}{3} \div \frac{1}{6}\right) = \frac{3}{4} x \left(\frac{2}{3} x \frac{6}{1}\right) =$

$\frac{3}{4} x\ 4 = 3$

$.2\ x\ (.12 \div .3) = .2\ x\ .4 = .08$

This coding should be used when two or more sequential operations are indicated in operational format.

0002 . . Topic I: Number Systems

1992 . . . C. Integers

1994 . . . . . . 1. Basic concepts

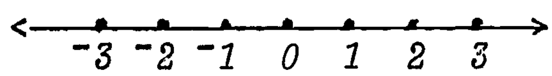
2000 . . . . a. Definition: set of integers

All of the numbers 0, ±1, ±2, ±3, ... form the set of integers.

The set that contains every natural number, its additive inverse, and zero is the set of integers.

2010 . . . . b. Developed from the number line

Ex.



2020 . . . . c. Developed from physical world situations

Ex. A thermometer uses a number line in vertical position. Show 10 degrees below zero on the number line. This is often written as $^{-}10^\circ$. *What does* $^{-}20^\circ$ *mean?* $^{+}40^\circ$?

Games like shuffleboard and monopoly use negative integers to indicate the player "owes" a score or play money. A disk may land on 10 OFF ($^{-}10$) *or a player may be in debt \$20* ($^{-}20$).

2030 . . . . d. Ordering: greater than; less than; equal to or not equal to; between

When the number line is in a horizontal position each numeral to the right of another numeral represents a greater number. Is $5 > 4$? *(Yes). Is* $^{-}5 > {}^{-}4$ *(No). Does* $^{-}4$ *lie to the right of* $^{-}5$? *(Yes). Is the number represented by* $^{-}5 <$ *the number represented by* $^{-}4$? *(Yes). What integer lies between* $^{-}6$ *and* $^{-}8$?

2040 . . . . e. Directed numbers: absolute value

Directed numbers are also called Positive and Negative numbers. Negative numbers are to be measured geometrically in a direction opposite to that in which positive numbers are measured.

(contd)

2040 (contd) *Ex.* *On a horizontal number line positive numbers are usually indicated to the right of zero. Then negative numbers are indicated to the left of zero. When the positive direction is determined on a vertical or slanted number line the negative direction will be opposite to it. Zero is the point from which other number points are established and is considered to be neither positive nor negative.*

The absolute value of a positive number is that number. Notation $|3| = 3$.

The absolute value of a negative number is the value of that number without regard to its sign. The absolute value of both $^{-}2$ *and* $^{+}2$ *is 2. Notation* $|^{-}2| = 2$.

On a number line the absolute value of a number is shown by its distance from zero without regard to the direction.

2042 . . . . . . 2. Operations

2044 . . . . a. Addition

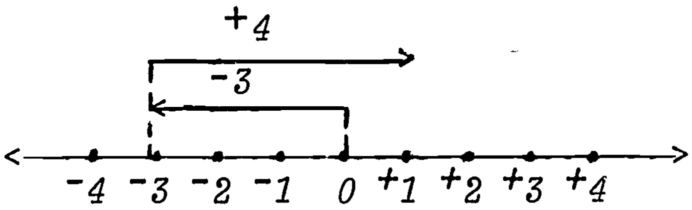
2046 . . . . 1) Properties

2050 . . . . a) Addition, a binary operation

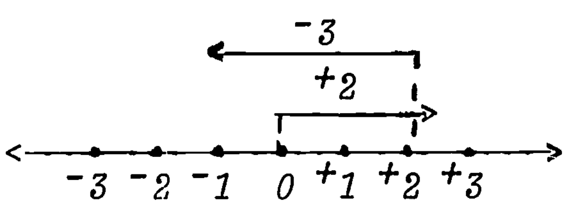
Binary - See 0110, page 3

2053 . . . . b) Addition developed from number line

Ex.



$^{-}3 + (^{+}4) = ^{+}1$



$^{+}2 + (^{-}3) = ^{-}1$

2055 . . . . c) Addition developed from physical world situations

Ex. *If the thermometer shows* $^{-}10°$ *and then rises* $^{+}15°$ *what is the temperature?* $^{-}10 + (^{+}15) = {}^{+}5$.

If you owe \$5 and pay \$3 what is your financial standing? $^{-}5 + (^{+}3) = {}^{-}2$ *(still owe \$2).*

If a submarine on the surface goes down 50 feet and then down 20 feet, what is its position? $^{-}50 + {}^{-}20 = {}^{-}70$ *(70 feet below sea level).*

2060 . . . . d) Closure, a property of addition

Closure - See 0140, page 4

2070 . . . . e) Commutativity, a property of addition

Commutativity - See 0150, page 4

Ex. $^{-}3 + (^{+}4) = 4 + (^{-}3)$

2080 . . . . f) Associativity, a property of addition

Associativity - See 0160, page 4

Ex. $(^{-}3 + 4) + (^{-}2) = {}^{-}3 + (4 + {}^{-}2)$

$$1 + {}^{-}2 = {}^{-}3 + 2$$

$$^{-}1 = {}^{-}1$$

2090 . . . . g) Zero, the identity element in addition

Identity - See 0170, page 5

Ex. $^{-}6 + 0 = 0 + {}^{-}6 = {}^{-}6$

$^{+}6 + 0 = 0 + {}^{+}6 = {}^{+}6$

2100 . . . . h) Additive inverse for addition

The additive inverse of any number is a second number which if added to the first number gives the sum of zero. For each integer $\underline{a}$ *the additive inverse is* $^{-}\underline{a}$.

Ex. $3 + (^{-}3) = 0$

$(^{-}4) + (^{+}4) = 0$

2110 . . . . 2) Addition computation

Ex. *See 2053.*

$^{+}3 + (^{+}8) = {}^{+}11$

$^{+}3 + (^{-}8) = {}^{-}5$

$^{-}3 + (^{+}8) = {}^{+}5$

$^{-}3 + (^{-}8) = {}^{-}11$

Do you see any pattern for these sums?

2112 . . . . b. Subtraction

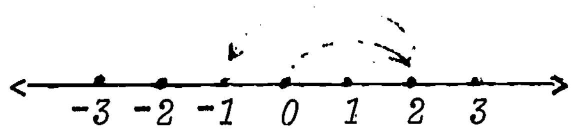
2114 . . . . 1) Properties

2120 . . . . a) Subtraction, a binary operation

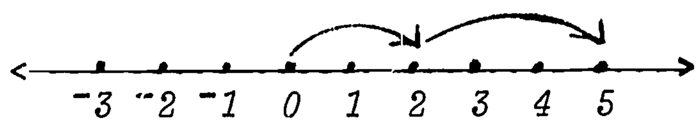
Binary - See 0110, page 3

2123 . . . . b) Subtraction developed from number line

Ex. $2 - {}^{+}3 = {}^{-}1$



Ex. $2 - (^{-}3) = {}^{+}5$



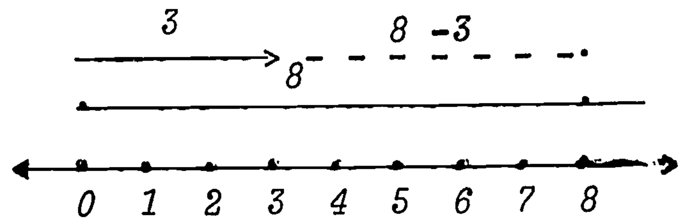
Subtracting a positive 3 on the number line was shown by movement to the left so subtracting a negative 3 must be shown by movement to the right of the positive 2.

A second explanation is possible when subtraction is considered as finding the difference between two numbers, or on the number line as finding distance between two number points.

(contd)

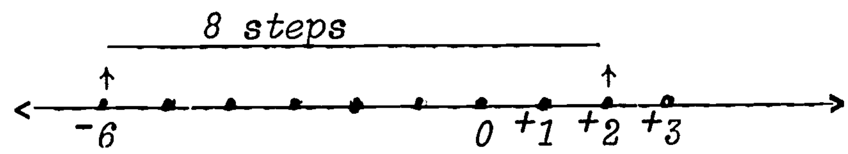
2123 (contd)

Ex. $8 - 3 = 5$



To find the difference between 3 and 8 on the number line we may ask, "How far is it from 3 to 8?" or think $8 - 3 = 5$. We may think it is 5 steps in the positive direction from 3 to 8.

Ex. $^{+}2 - (^{-}6) = {}^{+}8$



To find the distance from $^{-}6$, the known addend, to $^{+}2$, the sum move 8 steps from $^{-}6$ to $^{+}2$ in the positive direction showing the difference to be $^{+}8$.

Note: The distance and the direction are found in going from the known addend (subtrahend) to the sum (minuend).

2125 . . . . c) **Subtraction developed from physical world situations**

Ex. *If you had \$7 and bought something for \$10 you must subtract 10 from 7 to find your financial standing. Your standing is $7 - 10 = {}^{-}3$ or you will be \$3 in debt.*

Ex. *If you owed \$5 and subtracted \$2 of that debt, what is your financial standing? Taking away a debt is equivalent to adding the money so you may think $^{-}5 - (^{-}2) = {}^{-}5 + (^{+}2) = {}^{-}3$. You still owe \$3.*

2130 . . . . d) Subtraction, the inverse of addition

Inverse - See 0270, page 8

Ex. $^{+}5 - (^{-}2) = {}^{+}7$ because $^{+}7 + (^{-}2) = {}^{+}5$

$^{-}5 - (^{+}2) = {}^{-}7$ because $^{-}7 + (^{+}2) = {}^{-}5$

2140 . . . . e) Role of zero in subtraction

Ex. $^{-}3 - 0 = {}^{-}3$ / $^{+}4 - 0 = {}^{+}4$ — *Zero is the right hand identity element for subtraction*

$^{-}3 - (^{-}3) = 0$ / $n - n = 0$ — *Any number subtracted from itself is zero.*

$0 - (^{+}3) = {}^{-}3$ / $0 - (^{-}3) = {}^{+}3$ — *Any number subtracted from zero results in that number with its sign changed.*

2150 . . . . f) Closure, a property of subtraction

Closure - See 0140, page 4

2160 . . . . g) Noncommutativity, nonassociativity of subtraction

Commutativity - See 0150, page 4
Associativity - See 0160, page 4

Ex. Noncommutativity

$$^{-}2 - (^{-}5) \neq {}^{-}5 - (^{-}2)$$
$$^{+}3 \neq {}^{-}3$$

Ex. Nonassociativity

$$(^{+}3 - {}^{-}2) - {}^{+}5 \neq {}^{+}3 - (^{-}2 - {}^{+}5)$$
$$5 - 5 \neq 3 - (^{-}7)$$
$$0 \neq {}^{+}10$$

2170 . . . . 2) **Subtraction computation**

Ex. $^-3 - (^+4) = ^-7$

Think "How far is it from $^+4$ to $^-3$ and in what direction?"; or think $^-3 - (^+4)$ is equivalent to $^-3 + (^-4) = ^-7$.

$-3 - (^-4) = +1$

$^+3 - (^+4) = ^-1$

$^+3 - (-4) = ^+7$

2172 . . . . c. **Multiplication**

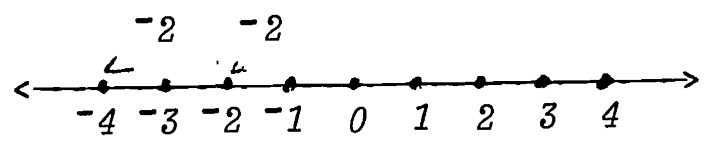
2174 . . . . 1) **Properties**

2180 . . . . a) **Multiplication, a binary operation**

Binary - See 0110, page 3

2183 . . . . b) **Multiplication developed from number line**

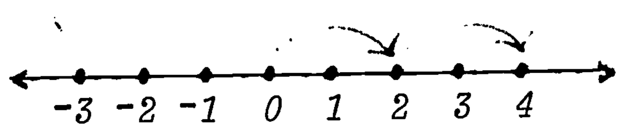
Ex.



$2 \times 2 = 4$ *(already known)*

$2 \times (^-2) = ^-4$ *(drawn on the number line)*

$-2 \times (+2) = -4$ *(see 2200, multiplication is commutative)*



$-2 \times (^-2)$ *must be drawn in a direction opposite to* $^+2 \times (^-2)$ *and equals* $^+4$.

2185 . . . . c) Multiplication developed from physical world situations

Ex. *If you have 2 x \$4 what is your financial standing?* $2 \text{ x } ({}^{+}4) = {}^{+}8$. *You have \$8.*

If you spend \$4 each week and receive no additional funds what is your financial standing after two weeks have passed? ${}^{+}2 \text{ x } ({}^{-}4) = {}^{-}8$. *You will have \$8 less than you have now.*

If you spend \$4 each week what was your financial standing 2 weeks ago? ${}^{-}2 \text{ x } ({}^{-}4) = {}^{+}8$. *You had \$8 more 2 weeks ago than you have now if you received no additional funds.*

2190 . . . . d) Closure, a property of multiplication

Closure - See 0140, page 4

2200 . . . . e) Commutativity, a property of multiplication

Commutativity - See 0150, page 4

Ex. $({}^{-}3) \text{ x } ({}^{+}8) = ({}^{+}8) \text{ x } ({}^{-}3)$

2210 . . . . f) Associativity, a property of multiplication

Associativity - See 0160, page 4

Ex. $({}^{-}3 \text{ x } {}^{+}2) \text{ x } {}^{+}5 = {}^{-}3 \text{ x} ({}^{+}2 \text{ x } {}^{+}5)$

$${}^{-}6 \text{ x } {}^{+}5 = {}^{-}3 \text{ x } {}^{+}10$$

$${}^{-}30 = {}^{-}30$$

2220 . . . . g) One, the identity element in multiplication

Identity element - See 0170, page 5

Ex. $1 \text{ x } {}^{-}5 = {}^{-}5 \text{ x } 1 = {}^{-}5$

$1 \text{ x } {}^{+}5 = {}^{+}5 \text{ x } 1 = {}^{+}5$

Multiplying by 1 leaves the number $({}^{+}5$ *or* ${}^{-}5)$ *unchanged.*

2230 . . . . 2) **Multiplication computation**

Ex. $\begin{array}{r} {}^{+}2 \\ x\ {}^{+}5 \\ \hline {}^{+}10 \end{array} \qquad \begin{array}{r} {}^{-}2 \\ x\ {}^{+}5 \\ \hline {}^{-}10 \end{array} \qquad \begin{array}{r} {}^{-}2 \\ x\ {}^{-}5 \\ \hline {}^{+}10 \end{array} \qquad \begin{array}{r} {}^{+}2 \\ x\ {}^{-}5 \\ \hline {}^{-}10 \end{array}$

What pattern do you see for these products?

2232 . . . . d. **Division**

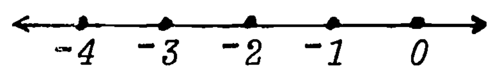
2234 . . . . 1) **Properties**

2240 . . . . a) **Division, a binary operation**

Binary - See 0110, page 3

2243 . . . . b) **Division developed from number line**

Ex.



If the space from 0 to $^{-}4$ *is divided by 2, what point is determined?* $\frac{^{-}4}{2} = {}^{-}2$. *Then* $\frac{^{-}4}{^{-}2}$ *must determine point* $^{+}2$. *Check both examples by multiplying.*

2245 . . . . c) **Division developed from physical world situations**

Ex. *The temperature fell from* $^{-}4$ *to* $^{-}12$ *degrees in 2 hours. What was the average change per hour?* $\frac{^{-}8}{2} = {}^{-}4$.

The temperature rose from $^{-}8$ *to* $^{-}2$ *in 3 hours. The temperature rose 6 degrees in 3 hours or* $\frac{6}{3}$ *or 2 degrees per hour.*

Ex. *The temperature is zero (0°) now. Two hours ago* ($^{-}2$) *it was 8 degrees below zero* ($^{-}8$). *What was the average change per hour?*

$$\frac{^{-}8}{^{-}2} = {}^{+}4$$

2250 . . . . d) Division, the inverse of multiplication

Ex. $\frac{^-10}{^-2} = {}^+5$ *because* $5 \times {}^-2 = {}^-10$

$\frac{^-10}{2} = {}^-5$ *because* ${}^-5 \times 2 = {}^-10$

$\frac{10}{^-2} = {}^-5$ *because* ${}^-2 \times {}^-5 = {}^+10$

2255 . . . . e) Role of one in division

Ex. ${}^-3 \div 1 = {}^-3$ *1 is the right hand identity element for division.*

$\frac{^-3}{^-3} = 1$ *Any number divided by itself is 1.*

$\frac{n}{n} = 1$

2260 . . . . f) Nonclosure, noncommutativity, nonassociativity of division

Closure - See 0140, page 4
Commutativity - See 0150, page 4
Associativity - See 0160, page 4

Ex. Nonclosure

$${}^-3 \div 2 = \frac{^-3}{2} \text{ (not an integer)}$$

Noncommutativity

$${}^-8 \div 2 \neq 2 \div {}^-8$$
$${}^-4 \neq \frac{^-1}{4}$$

Nonassociativity

$$({}^-8 \div 4) \div 2 \neq {}^-8 \div (4 \div 2)$$
$${}^-2 \div 2 \neq {}^-8 \div {}^+2$$
$${}^-1 \neq {}^-4$$

2270 . . . . 2) Division computation

Ex. $\frac{^-12}{4} = -3$ Check $^-3 \times 4 = {}^-12$

$\frac{^-12}{^-4} = +3$ Check $3 \times {}^-4 = {}^-12$

$\frac{12}{^-4} = -3$ Check $^-3 \times -4 = +12$

$\frac{^+12}{^+4} = +3$ Check $^+3 \times {}^+4 = {}^+12$

Do you see any pattern for these factors?

0002 . . Topic I: Number Systems

2500 . . . D. Negative rationals (fractional numbers)

2510 . . . . . . 1. Basic concepts

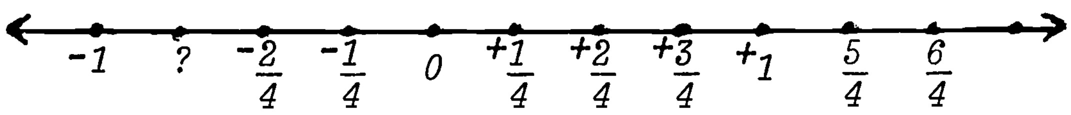
2520 . . . . a. Definition for a set of negative rationals

Negative rationals are numbers which can be expressed as a ratio of two integers as a/b where b *cannot be 0 and where either* a *or* b *is negative.*

Ex. $^-3/4$; $\frac{3}{-4}$ *Both fractions may be written as* $\frac{^-3}{4}$.

2530 . . . . b. Developed from number line

Ex.



2600 . . . . . . 2. Computation

Ex.

$$\frac{^-1}{2} + \frac{^+3}{4} = \frac{^-2}{4} + \frac{^+3}{4} = \frac{^+1}{4}$$

$$\frac{^-1}{2} - \frac{^-3}{4} = \frac{^-2}{4} + \frac{^+3}{4} = \frac{^+1}{4}$$

$$\frac{^-1}{2} x \frac{^-3}{4} = \frac{^+3}{8}$$

$$\frac{^-1}{2} \div \frac{^-3}{4} = \frac{^-1}{2} x \frac{4}{-3} = \frac{^-4}{-6} = \frac{2}{3}$$

Computation follows the patterns developed for positive fractions except that the signs used follow the patterns established for computation with integers.

0002 . . Topic I: Number Systems

2700 . . . E. Natural Numbers (counting numbers)

2710 . . . . 1. Basic concepts

2720 . . . . a. Definition for a set of natural numbers

The set of natural numbers is shown as

$N = \{1, 2, 3, \cdots\}$

2730 . . . . b. Relation to set of whole numbers, nonnegative rationals, integers, negative rationals

Ex. $N = \{1, 2, 3, \cdots\}$ *set of natural numbers*

$W = \{0, 1, 2, \cdots\}$ *set of whole numbers*

$I = \{\cdots {}^-2, {}^-1, 0, {}^+1, {}^+2, {}^+3, \cdots\}$ *set of all integers*

NNR are such fractions as $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{12}{5}$, $\frac{6}{6}$, *etc.*
They cannot be written with set notation because density is a property of rational numbers.

NR are fractions such as $\frac{{}^-10}{3}$, $\frac{{}^-2}{5}$, $\frac{{}^-6}{6}$, *etc.*
They cannot be written with set notation because density is a property of rational numbers.

The set of natural numbers and the set of whole numbers are subsets of the set of integers. The set of non-negative rational numbers and the set of negative rational numbers are subsets of the set of all rational numbers. The set of rational numbers is developed from the set of natural numbers.

$N \subset W$

$W \subset I$

$I \subset R$

$NNR \cup NR$ = *Rationals*

R is a subset of the Real Numbers.

(contd)

2730 (contd)

$$\textit{Natural Numbers} \subset \underbrace{\textit{Natural Numbers} \cup \textit{zero}}_{\textit{Whole Numbers}} \subset$$

$$\textit{Integers} \subset \textit{Rationals} \subset \textit{Reals} \subset \textit{Complex}$$

Natural Numbers (positive integers)
Zero
Negative Integers
→ *Integers* and *Fractional Numbers (positive and negative)* → *Rationals* and *Irrationals* →

Reals and *Imaginary Numbers* → *Complex Numbers*

2992 . . Topic II: Numeration and Notation

3000 . . . A. Difference between number and numeral

We shall consider number to be the property shared by a collection of matched sets, as 2 is the cardinal number of the sets {X, Y} and {A, B}. We shall consider numerals as the names for numbers, 5, V, 100, etc. A number is an idea, abstract, and cannot be written or seen. A numeral is a symbol for the number, concrete, and can be written and seen.

3002 . . . B. Different numerals for the same number (renaming)

3010 . . . . . . 1. Expanded notation for whole numbers

Expanded notation is notation using numerals showing the place value of each digit.

Ex. $874 = 800 + 70 + 4$ *or* 874

$874 = 8$ *hundreds* $+ 7$ *tens* $+ 4$ *ones*

Polynomial form

$(8 \times 100) + (7 \times 10) + (4 \times 1)$

or

$(8 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)$

3015 . . . . . . 2. Expanded notation for nonnegative rationals (fractions)

Ex. $3\frac{1}{2} = 3 + \frac{1}{2}$

$.35 = .3 + .05$

$.35 = \frac{3}{10} + \frac{5}{100}$

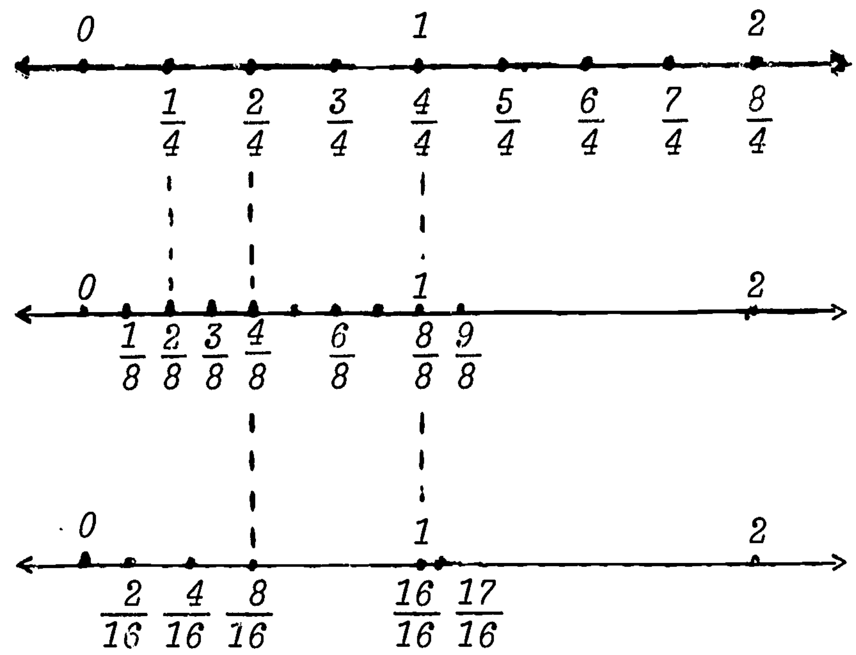
$.35 = [(3 \times \frac{1}{10^1}) + (5 \times \frac{1}{10^2})]$

$56.63 = 50 + 6 + .6 + .03$

$56.63 = [(5 \times 10) + (6 \times 1) + (6 \times \frac{1}{10}) + (3 \times \frac{1}{100})]$

3020 . . . . . . 3. Equivalent common fraction notation

Ex.



The fraction (numeral) $\frac{2}{4}$ *may be renamed as* $\frac{4}{8}$, $\frac{8}{16}$.

$\frac{1}{2}$ *may be renamed as* $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$, *etc.*

3025 . . . . . . 4. Equivalent mixed numeral notation

Ex. $1\frac{3}{4} = \frac{7}{4}$, $4\frac{1}{3} = \frac{13}{3}$, $\frac{14}{3} = 4\frac{2}{3}$

3030 . . . . . . 5. Equivalent decimal fraction notation with terminating decimals

A terminating decimal has a finite number of digits.

Ex. $.75 = \frac{3}{4}$

$\frac{3}{8} = .375 = \frac{375}{1000}$

$\frac{1}{2} = .5$

$3\frac{1}{2} = 3.5$

3033 . . . . . . 6. Equivalent decimal notation with repeating decimals

A repeating decimal numeral has an initial pattern of digits followed by a continuous repetition of a single digit or a pattern of digits.

Ex. $\frac{1}{6} = .1666\cdots$. $\frac{5}{12} = .41666\cdots$. $\frac{1}{7} = .1428571428571\cdots$

3035 . . . . . . 7. Equivalent per cent notation

Ex. $25\% = .25 = \frac{25}{100} = \frac{1}{4}$

$\frac{3}{8} = .375 = 37\frac{1}{2}\%$

$2.5 = 2\frac{1}{2} = 250\%$

3040 . . . . . . 8. Other names for a number

Use this code largely for other names for natural numbers, whole numbers, or integers. Use 3020, 3025, 3030, 3033, and 3035 for other names for rational numbers.

Use when not classified in 3010-3035.

If the purpose of the lesson is the development of basic facts, do not code 3040.

Ex. Some other names for 6 are:

2×3	$12 \div 2$
$2 + 4$	$1 + 1 + 4$
$7 - 1$	$\frac{1}{2}$ of 12

3042 . . . C. Place value in base ten

3050 . . . . . . 1. Reading and/or writing words or numerals for the ten basic symbols (0-9)

Ex. See any Grade I textbook showing how numerals 0-9 are written. Reading numerals or words may occur in exercises such as: Make a mark on the set showing 5 (five) members.

3070 . . . . . . 2. Units, tens (10-99)

Ex. 12 means 1 ten and 2 ones.

Use code 3070 when place value is being emphasized.

3080 . . . . . . 3. Beyond tens

Ex. 103 means 1 hundred, no tens, and 3 ones.

Use code 3080 when place value is being emphasized.

3090 . . . . . . 4. Commas to separate into periods

Ex.

millions *thousands* *units or ones*

3, 210, 000

3100 . . . . . . 5. Rounding numbers

Ex. *37 rounded to the nearest 10 is 40.*
673 rounded to the nearest 100 is 700.
428 rounded to the nearest 100 is 400.

35 may be rounded to the nearest 10 as 30 or 40. The text used will determine the policy.

3110 . . . . . . 6. Decimal fractions

Decimal fractions may be considered another way of naming rational numbers which in fraction form have some power of 10 as a denominator.

Ex. $\frac{7}{10} = .7$ $\frac{23}{1000} = .023$

···1 2 3 . 4 5 6 7 8···

etc. *hundreds* *tens* *ones* *tenths* *hundredths* *thousandths* *ten thousandths* *hundred thousandths* *etc.*

3120 . . . . . . 7. Exponential notation

Exponential notation is notation using numerals which have exponents, small numerals written to the right and above a base numeral. An exponent may be either positive or negative. It may also be a fraction.

Ex. $874 = (8 \times 10^2) + (7 \times 10^1) + (4 \times 10^0)$

$= (8 \times 10 \times 10) + (7 \times 10) + (4 \times 1)$

$3^4 = 3 \times 3 \times 3 \times 3$

$3^{-2} = \frac{1}{3^2} = \frac{1}{3 \times 3}$

Use code 3120 when notation of exponents is developed. Operations with exponential notation are coded 0520, 0670.

3130 . . . . . . 8. Scientific notation

Scientific notation is of the form 2.69×10^3. For any numeral the decimal point is placed immediately to the right of the first nonzero digit and the number is then multiplied by the integral power of 10 that would have the effect of shifting the decimal point to its original position.

Ex. 25 *is written in scientific notation as* 2.5×10^1

$236 = 2.36 \times 10^2$

$.000236 = 2.36 \times 10^{-4}$

3140 . . . D. **Historical development of number concepts**

In primitive times man may have noted the number of animals he killed by dropping a stone on a pile or making a mark for each on a rock realizing that he had more than one animal and later realizing that he needed names for this counting. (See any History of Mathematics text for detailed development).

3150 . . . E. Historical systems of notation (nonplace or place value)

Historical systems of numeration are considered to be those used before the Hindu-Arabic system; such as the system of Roman Numerals or Egyptian numerals.

Ex. XV in Roman numerals means 15 in Hindu-Arabic symbols.

𓍢 in Egyptian symbols means 100 in Hindu-Arabic symbols.

/// often marked on the rocks or sand means 3 in Hindu-Arabic symbols.

3160 . . . F. Working with nondecimal place value systems (other number bases)

Nondecimal place value numeration systems are built on bases other than 10 but still use place value.

Ex. 413_{five} *or 413 (base five) means* $(4x5^2)+(1x5^1)+(3x5^0)$

413_{eight} *or 413 (base eight) means* $(4x8^2)+(1x8^1)+(3x8^0)$

413 in any base five or larger means $4 \times base^2 + 1 \times base^1 + 3 \times base^0$.

Note: Since 4 is used in the numeral the base must be at least as large as five.

3992 . . Topic III: Sets: physical and abstract

3994 . . . A. Description of sets

Any collection of individual objects into a whole. A set only exists if it is well-defined; that is, one is able to tell whether an object is a distinct member of the set.

A set is usually indicated by braces {xx} or closed curves xx, $\boxed{xx}$.

Ex. Physical objects - a set of dishes
a set of dominoes

Abstract - the set of whole numbers

4000 . . . B. Set members or elements

Members - Each object in the set (collection) is a member or element of the set.

Ex. The set {□, ○, ◺, ▭} *has 4 members or elements.*

The square, circle, triangle, rectangle are elements of the set.

In the set {6, 7, 8, 9} each number is a member or an element of the set.

4004 . . . C. Kinds of sets

4010 . . . . 1. Equivalent (one-to-one correspondence)

Equivalent sets have the same number of members but not necessarily the identical members. Members of equivalent sets can be paired in one-to-one correspondence.

Ex. A = {▽ ✳ ⬭}
↕ ↕ ↕
B = {□ ○ ⇨}

A and B are equivalent sets since each has 3 elements or since the elements can be shown in one-to-one correspondence.

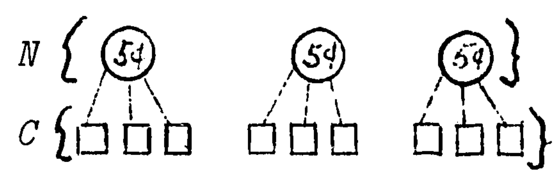
4030 . . . . 2. Non-equivalent (general)

Non-equivalent sets do not have the same number of members and cannot be paired in one-to-one correspondence.

Ex. {□, ○} *and* {▽, ▯, ⬡} *are <u>not</u> equivalent sets.*

4035 . . . . 3. Non-equivalent (one-to-many correspondence)

Ex. If one nickel will buy 3 pieces of candy, then two nickels will buy 6 pieces of candy, etc.



In the drawing one element from the set of nickels is matched with three elements from the set of pieces of candy.

4037 . . . . 4. Equal (identical)

Equal sets have exactly the same members. They will then have the same number of members and are therefore also equivalent.

Ex. {□, ○, ◺} *and* {○, ◺, □} *are equal sets since the members or elements are identical though not shown in the same order.*

4040 . . . . 5. Unequal

Unequal sets do not have identical members or elements though they may have the same number of elements in which case they are equivalent sets.

Ex. *See 4010, page 61.*

{□, ○, ◺} *and* {◇, ▱, ○} *are unequal since elements are not identical.*

{1,2,3,4,5} and {12,3,4,5} are unequal.

4060 . . . . 6. Subsets

If each member of a set B is a member of a set A, we say that B is a subset of A.

Ex. $A = \left\{\begin{matrix}\textit{Set of all pupils,} \\ \textit{boys and girls, in} \\ \textit{the room}\end{matrix}\right\}$ $B = \left\{\begin{matrix}\textit{Set of all boys} \\ \textit{in the room}\end{matrix}\right\}$

B is a subset of A since all the boys belong to the set of all the pupils.

$N = \{2,4,6\}$. *The subsets of N are* $\{2\}$; $\{4\}$; $\{6\}$; $\{2,4\}$; $\{2,6\}$; $\{4,6\}$; $\{\ \}$; $\{2,4,6\}$. *The symbol to indicate a subset is* $\subset$. $B \subset A$ *is read B is a subset of A.*

Note: A subset may be removed from a set. *See 0250, page 7.*

A set may be partitioned into equivalent subsets. *See 0540, page 15.*

4070 . . . . 7. The empty set

The empty set has no members or elements. The set of students with four legs is an empty set. The cardinal number of the empty set is zero. $\{\ \}$ *is one symbol for the empty set. The empty set is a subset of every set.*

4090 . . . . 8. Disjoint

Disjoint sets are sets which have no elements in common.

Ex. $A = \{○, □\}$ $B = \{△, ▭\}$ *A and B are disjoint sets.*

$A \cap B = \{\ \}$

$C = \{1,3,0\}$ $D = \{7,4\}$

$C \cap D = \{\ \}$ *or* $\emptyset$

4093 . . . . 9. Union of sets

The union of two sets, denoted by the symbol $\cup$, *is the set of all elements belonging to either of the two sets or to both of them. Elements common to both sets are not repeated when naming the set union.*

Ex. $A = \{1,3,5\}$ $B = \{2,3,5,7\}$ $A \cup B = \{1,2,3,5,7\}$

4095 . . . . 10. Intersection of sets

The intersection of two or more sets, denoted by the symbol, $\cap$, is the set of elements common to both or all sets.

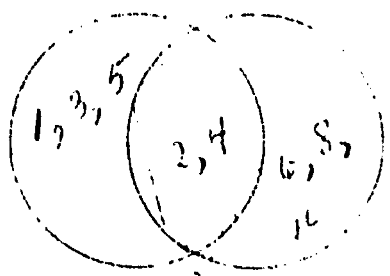
Ex. *If $A = \{1,3,5,7\}$ and $B = \{5,7,9,11\}$ then A and B are intersecting sets.*

$A \cap B = \{5,7\}$; read A intersection B, is the set $\{5,7\}$ where 5 and 7 are the elements common to both sets.

4097 . . . . 11. Venn diagrams

Venn diagrams are diagrams which use overlapping or intersecting circles to show relationships between sets.

Ex. A



B

$A = \{1,2,3,4,5\}$ *$B = \{2,4,6,8,10\}$*

$A \cap B = \{2,4\}$ The shaded area shows the intersection.

Use code 4097 only if Venn diagrams are identified by the authors.

4100 . . . . 12. Finite

Finite sets are sets which can be defined by counting, with the counting coming to an end. There is a whole number that identifies the number of members.

Ex. *$A = \{2,4,6\}$ all the members are identified.*

$B = \{2,4,6,\cdots,20\}$ is a finite set since all the members can be identified by counting.

4110 . . . . 13. Infinite

Infinite sets are those which cannot be named by counting, with the counting coming to an end.

Ex. *$\{1,2,3,4,\cdots\}$ The set of natural numbers is an infinite set.*

The three dots indicate that the set is infinite and that you may write other elements which continue the pattern indefinitely.

4120 . . . . . . 14. **Universal and complement**

A universal set is the set containing all elements under consideration and is usually designated by U or I.

Ex. $U = \{\text{all states in the U.S.}\}$ $A = \{\text{Iowa, Minn., New York}\}$

$A \subset U$

If I is the universe under consideration, and A any subset of I, then the set composed of all the elements of I that are not in A is called the complement of A, and is usually designated by A'.

Ex. If I represents all the pupils in a room and A represents all the pupils with blue eyes, then the set complement of set A is A' or all the pupils who do not have blue eyes.

4125 . . . . . . 15. **Solution sets**

The set of elements which when used to replace the variable(s) in an open sentence makes it a true sentence or the set of all numbers that are solutions for a number sentence is called a solution set.

Ex. $3 \times n > 60$. *The solution set for n, an integer, is* $\{21, 22, 23, \cdots\}$

If $A = \{x | x > 5\}$ *in the universe of whole numbers, then the solution set* $S = \{6, 7, 8, \cdots\}$

4160 . . . . . . 16. **Cartesian product sets (cross products)**

The set of all pairs of elements from set A and set B such that the first element in the pair is from set A and the second from set B.

Ex. $A = \{x, y\}$, $B = \{1, 2, 3\}$. *The product set is* $\{(x,1), (x,2), (x,3), (y,1), (y,2), (y,3)\}$. *The number of pairs in the product set is the cardinal number of the Cartesian product. In this case it is 6.*

Note: See 0390 for use with introduction of multiplication of whole numbers.

4992 . . Topic IV: Geometry

5000 . . . A. Intuitive concepts of geometric figures and ideas

5010 . . . . . . 1. Geometric figures in environment

Shapes such as circles, squares, and triangles become familiar through pictures or objects seen in room, on trips, at home.

Note: Use this code in introductory lessons to geometric figures where a variety of familiar shapes are used.

5020 . . . . . . 2. Geometric designs or patterns (sequences)

Ex. □—○□—○— ? — ?

Continue the pattern.

5030 . . . . . . 3. Spatial relations without measurement (size, position)

Ex. Mark an x on the larger ball.
Mark an x on the largest ball.
Mark an x above (below) the doll.

5040 . . . . . . 4. Two dimensional figures (plane)

Plane figures are two dimensional. They are measured only by their length and width.

Ex. Some models of plane figures are the surfaces of floors, window panes and doors.

5050 . . . . . . 5. Three dimensional figures (solid)

Figures in space are three dimensional. They are measured by length, width and height.

Ex. Some models of three dimensional figures are a cereal box, a tin can, and an ice cream cone.

5060 . . . . 6. **Curves: simple, closed, open**

A curve in a plane may be thought of as a set of points represented by a drawing made without lifting the pencil from the plane.

Ex.

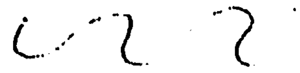


Figure A

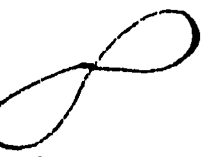


Figure B

Figure C

A simple curve does not pass through any point twice. Figure A (above) is a simple curve. Figure B (above) is not a simple curve.

A closed curve begins and ends at the same point.

Ex.

Figure D

An open curve does not end at the beginning point.

Ex.

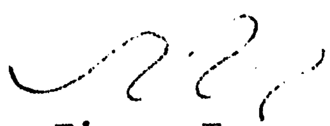


Figure E

Figure F

A simple closed curve is undefined but may be thought of as beginning and ending at the same point so that in tracing the complete curve no other point is encountered twice. The continuous mark may consist of straight line segments as well as curved segments.

Ex.

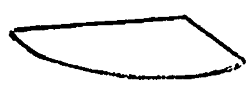


Figure G

Figure G is a simple closed curve. Figure B (above) is a closed curve but <u>*not*</u> *a simple closed curve.*

5070 . . . . 7. **Regions: interior, exterior**

A simple closed curve separates the plane into 3 sets of points.

all points outside the curve (exterior region), all points inside the curve (interior region), and all points on the curve (boundary). The boundary has no points in common with either its interior or exterior regions.

(contd)

5070 (contd) *Area is the measurement of the interior region and boundary of plane figures.*

Note: Some elementary textbooks defined region as the union of a simple closed curve and its interior. Instead of interior region and exterior region as defined in the Content Authority List, the three sets of points are called: the curve, the interior, and the exterior.

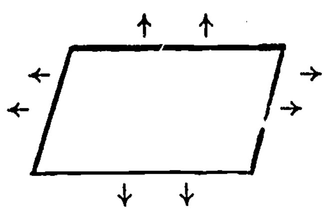
See McGraw-Hill, Book 1, p.56,107.
SMSG, Book 4, p.473.

5080 . . . . . . 8. **Representations of point, line, plane, space**

A point is a concept which like a number exists only in the mind. As a numeral represents a number, a dot (·) represents a point. The tip of a pen or the sharp end of the lead in a pencil would suggest a point.

⟷ *A line is a set of points extending infinitely in opposite directions.*

Ex. The marks where the walls meet the ceiling make you think of lines though they do not go on forever.



A flat surface suggests a plane. The set of points in a plane extends infinitely in all directions.

Ex. Some things in the room which make you think of a plane are desk top, floor, a piece of paper. Perhaps the shadow is the best model since it is flat and has no thickness.

Space is the set of all points. We think of geometric figures like your book, or ball, or box of dominoes as space figures.

5082 . . . B. **Concepts of geometric figures and ideas explored in depth**

Study may include definitions for geometric figures, listing their characteristics or drawing conclusions from experiments. See 5202 for construction code.

5090 . . . . 1. Point

A point is a location. It has no size. The intersection of two lines determines a point. Between any two points there is always another point.

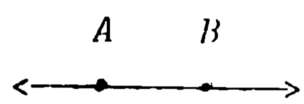
5100 . . . . 2. Line

A line is a set of points which extend infinitely in opposite directions.

For any two points there is only one line which passes through both of them.

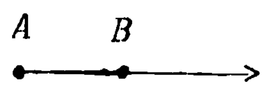
Two different lines intersect in only one point.

5101 . . . . 3. **A line segment is an infinite set of points (a subset of a line) with two points indicating endpoints**

Ex. 

$\overline{AB}$ is a picture (model) of a line segment. A wire stretched between two posts is a model of a line segment.

5103 . . . . 4. **A ray is an infinite set of points (a subset of a line) with only one endpoint. A second point in the ray helps to read it**

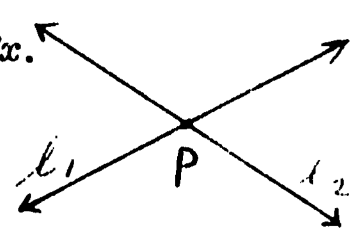
Ex. 

$\overrightarrow{AB}$ is a picture of a ray.

A beam of light coming from the sun is often called a light ray.

5105 . . . . 5. **Related lines: intersecting, parallel, skew, oblique, ...**

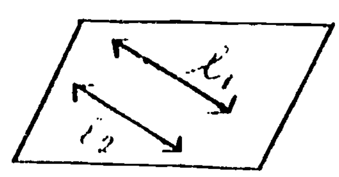
Lines drawn through a common point are called intersecting lines.

Ex. 

ℓ_1 and ℓ_2 are intersecting lines through point P. How many lines can intersect at point P? (infinite number).

Parallel lines are lines lying in the same plane and having no point in common or all points in common.

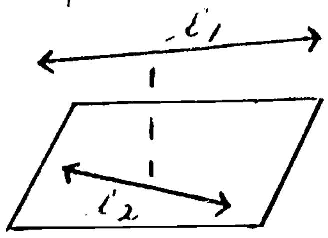
Ex. Two rails of a railroad track represent parallel lines.



(contd)

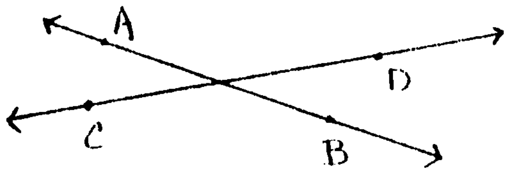
5105 (contd) *Skew lines are lines that have no point in common and do not lie in the same plane.*

Ex. *A string stretched across the floor and a string stretched by two pupils at waist height so that it is not parallel to the first string represent skew lines.*



Oblique lines in a plane are lines which are neither parallel nor perpendicular. When they meet they will form an angle greater or less than a right angle. $\overleftrightarrow{AB}$ *and* $\overleftrightarrow{CD}$ *are oblique to each other.*

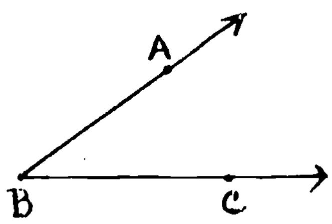
Ex.



5115 . . . . . . 6. Angles

An angle is the union of two rays with the same endpoint.

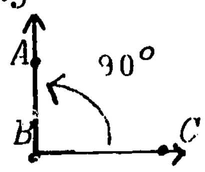
Ex.



5125 . . . . . . 7. Kinds of angles

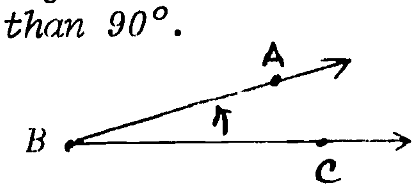
A right angle is an angle whose measure is 90°.

Ex.



An acute angle is an angle whose measure is greater than 0° and less than 90°.

Ex.



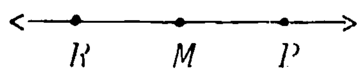
An obtuse angle is one whose measure is greater than 90° and less than 180°.

Ex.

(contd)

5125 (contd) *A straight angle is the union of two opposite rays having the same endpoint.*

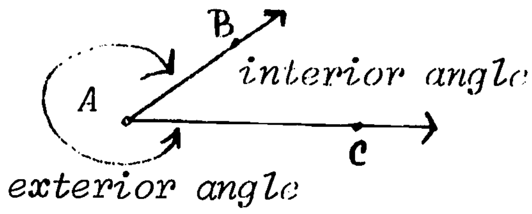
Ex.



5140 . . . . 8. Regions: interior, exterior

See 5070, page 67, for regions of plane figures.

The regions of an angle are:



The interior region of space figures is measured in volume.

5143 . . . . 9. Planes: two dimensional figures

Every plane contains at least three points not in a straight line (not collinear). A plane is a surface such that a straight line joining any two of its points lies entirely in the surface. A plane is a flat surface. The intersection of two planes is a line. An unlimited number of planes can pass through a line determined by two points. A plane figure has two dimensions, length and width.

5145 . . . . . 10. Polygons (plane figures)

A polygon is a closed, plane figure composed of N straight lines; N being ≥ 3.

Use for the properties of polygons in general such as number of vertices, diagonals.

Use this code when more than two types of polygons are considered in the same lesson.

5150 . . . . a. Triangles

See 5280 and 5290 for perimeter and area.

A triangle is a polygon of three sides.

An equilateral triangle is one whose three sides are equal in length (or measure).

(contd)

5150 (contd)

An isosceles triangle is a triangle with at least two sides or two angles equal in measure.

A scalene triangle is a triangle with no two sides equal in measure.

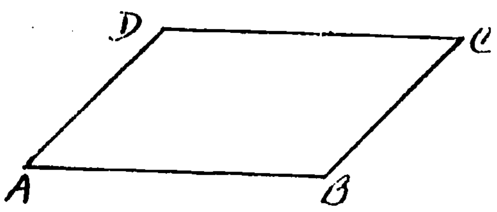
A triangle having one angle whose measure is 90° is a right triangle.

5160 . . . . b. Quadrilaterals

Quadrilaterals are polygons having four sides.

A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

Ex.



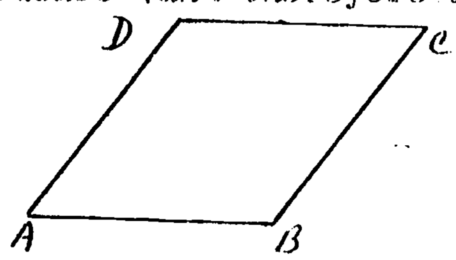
AD // CB
DC // AB

A rectangle is a parallelogram with one right angle (and therefore with four right angles).

A square is a rectangle with two adjacent sides equal in measure (and therefore with all four sides equal).

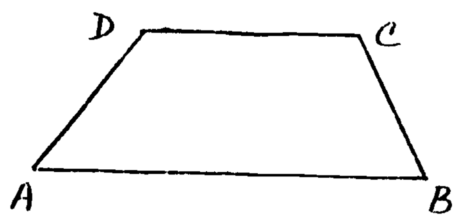
A rhombus is a parallelogram with two adjacent sides equal in measure (and therefore with all four sides equal).

Ex.



A trapezoid is a quadrilateral with one and only one pair of parallel sides.

Ex.



AB // CD

5170 . . . . c. Other polygons

A pentagon is a polygon with five sides.

A hexagon is a polygon with six sides.

An octagon is a polygon with eight sides.

5172 . . . . . 11. Curves

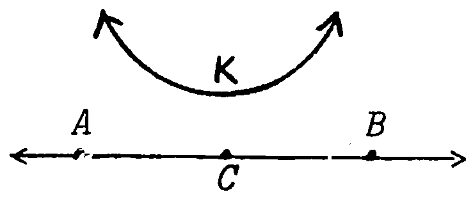
5174 . . . . a. Simple, closed, open

See 5060, page 67

5176 . . . . b. Convex, concave

A curve is said to convex toward a point or line if it curves toward the point or line

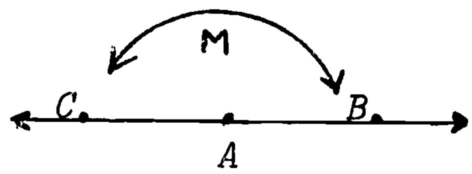
Ex.



Curve K is convex toward $\overleftrightarrow{AB}$ and point C.

A curve is said to be concave toward a point or a line if it is hollow toward the point or line.

Ex.

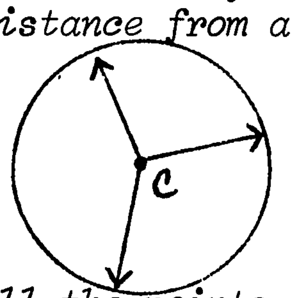


The curve is concave toward $\overleftrightarrow{CB}$ and point A.

5180 . . . . . 12. Circles

A circle is the set of all points in a plane which are the same distance from a given point called the center of the circle.

Ex.



All the points on the curve are equidistant from the center C.

The radius, diameter and a chord of a circle have special characteristics.

A circle separates the plane into interior and exterior regions.

Central angles cut off arcs and sectors of the circle.

See codes 5280 and 5290 for coding circumference and area of circles.

5183 . . . . . 13. **Space: three dimensional figures**

Space is the set of all points.

A figure in space separates all points into two regions, exterior and interior. A space figure has three dimensions, length, width and height.

Ex. The interior region of a sphere is the union of its center point and all points whose distances from the center point are less than the radius. The exterior region is all other points not on the surface of the sphere.

Some space figures are the cone, the sphere, the cylinder and prisms.

5185 . . . . . 14. **Space figures: three dimensional figures**

5186 . . . . a. **Pyramid**

A pyramid is formed by the union of triangular regions and a polygon which forms the base. If the base is a triangle or a square, the pyramid is a triangular or square pyramid.

5188 . . . . b. **Prism**

A (right) prism is formed by two congruent regions (the bases) whose matching sides are parallel and are joined by rectangular regions.

A cube is a prism formed by the union of six congruent square regions with common sides.

A rectangular prism is formed by the union of six rectangular regions with common sides.

A triangular prism is formed by the union of two triangular regions and three rectangular regions with common sides.

5191 . . . . c. **Cylinder**

A (right circular) cylinder is formed by the union of two congruent circular regions (bases) and the curved surface that intersects the perimeters of the bases and is perpendicular to them.

5192 . . . . d. **Cone**

A (right circular) cone is the union of a circular region (the base) and the surface formed by the union of line segments with one common endpoint (the apex) and the other endpoints on the perimeter of the base. A line segment that connects the center of the base and the apex is perpendicular to the base.

5194 . . . . e. **Sphere**

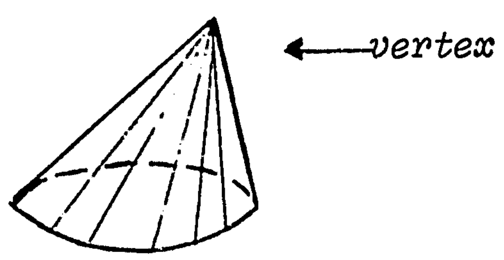
A sphere is the set of all points in space that are a fixed distance (the radius) from a given point called the center.

5195 . . . . . 15. **Conic sections other than circles; the ellipse, parabola, and hyperbola**

The ellipse, parabola, and hyperbola are formed by intersections of a conical surface and a plane.

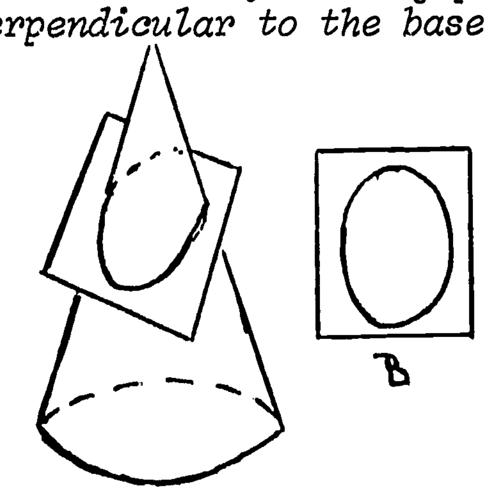
A cone is a solid consisting of the union of all line segments connecting a circular plane region with a point called the vertex, not in that plane. The surface of this figure is called the conical surface.

Ex.



Ellipse - A simple closed curve formed by plane B neither parallel nor perpendicular to the base.

Ex.



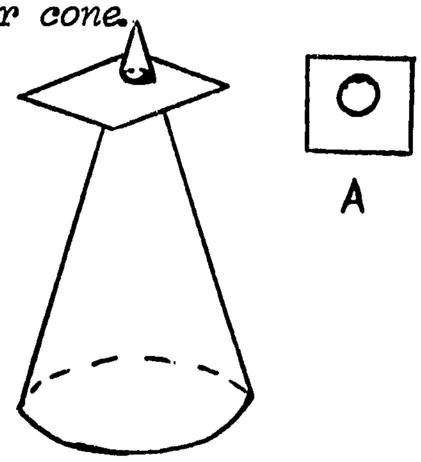
An equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(contd)

5195 (contd) *Circle* - See 5180.

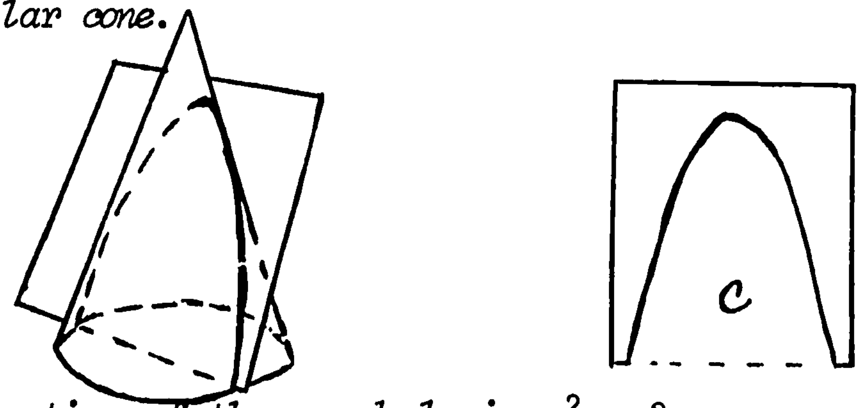
A simple closed curve formed by plane A parallel to base of a right circular cone.

Ex.



Parabola - *An open curve formed by plane C parallel to any element of a right circular cone.*

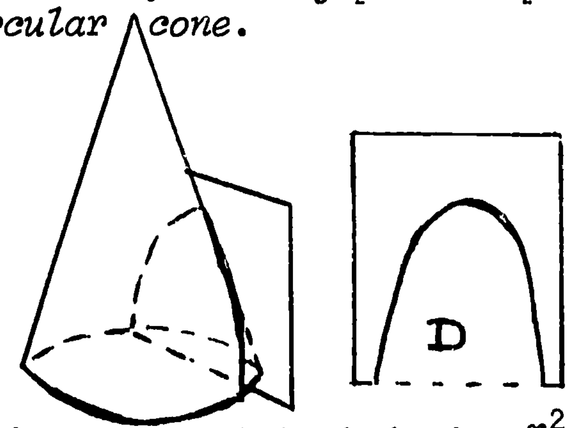
Ex.



An equation of the parabola is $y^2 = 2px$

Hyperbola - *An open curve formed by plane D perpendicular to the base of a right circular cone.*

Ex.



An equation of the hyberbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

5202 . . . C. **Constructions**

Only recognized geometric constructions will be coded 5210, 5220, and 5230. Drawing geometric figures will be coded under 5080.

5210 . . . . 1. **Line constructions (one dimensional figures)**

5220 . . . . 2. **Two dimensional figures (plane figures)**

5230 . . . . 3. **Three dimensional figures (figures in space)**

5232 . . . D. Metric Geometry

5234 . . . . . . 1. Comparing sizes, shapes, distances (including latitude, longitude)

5240 . . . . a. Congruency

Congruency is the property of the relation of two geometric figures having the same size and shape.

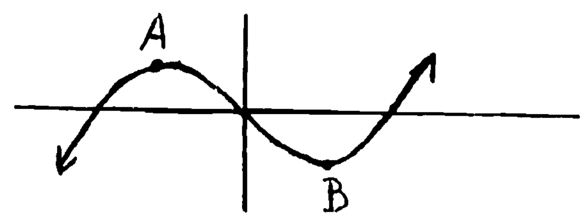
Angles are congruent when their measures are equal.

Two dimensional figures are congruent when their corresponding sides and corresponding angles are equal in measure.

5245 . . . . b. Symmetry

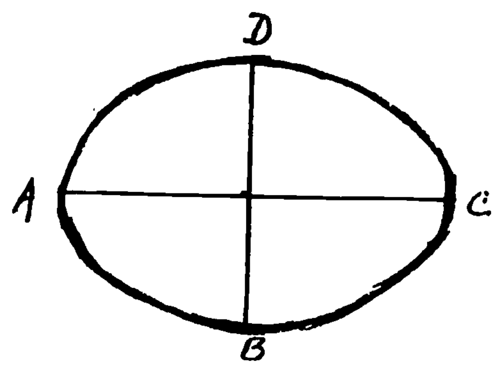
A geometric configuration is said to have symmetry with respect to a point, a line, or a plane when for every point on the figure there is another point such that the pair correspond with respect to the point, the line, or the plane.

Ex.



Points A and B are symmetric to the origin, a point.

Ex.



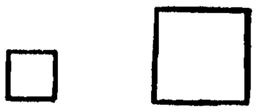
The ellipse ABCD is symmetric to the line AC and to the line DB.

5250 . . . . c. Similarity

Similar geometric figures have the same shape, but not necessarily the same size.

(contd)

5250 (contd) *Ex.*



Similar polygons have the angles of one equal in measure to the corresponding angles of the other and the corresponding sides in proportion.

5255 . . . . d. Similarity: scale drawing

In a scale drawing all distances are in the same ratio to the corresponding distances on the original figure.

5258 . . . . . . 2. Measurement of geometric quantities

5260 . . . . a. Line segments with ruler and/or compass or other measuring device

5270 . . . . b. Angles with protractor and/or compass or other measuring device

5280 . . . . c. Perimeter or circumference of simple closed curves

5290 . . . . d. Area of plane figures

Include lessons finding surface areas of solids.

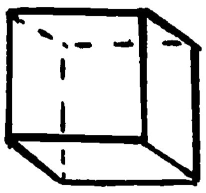
5300 . . . . e. Volume of solids

5400 . . . E. Operations with geometric figures

5410 . . . . . . 1. Union

Union - See 4093, page 63

Ex. The union of six plane rectangular regions forms a rectangular solid.



5420 . . . . . . 2. Intersection

Intersection - See 4095, page 64

The faces of a rectangular solid intersect in line segments.

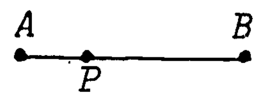
The base and the conical surface of a right circular cone intersect in a circle.
See Silver-Burdett, Book 8, p. 296-297.

5500 . . . F. Other Topics

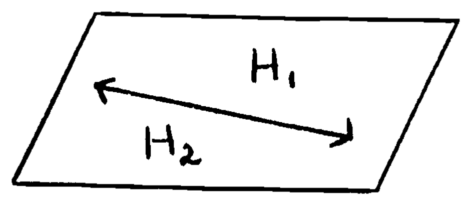
5510 . . . . . . 1. **Separation of sets of points**

Marking off points, in some manner, to show the relation between groups or sets is called separation of sets of points.

Ex. *The point P partitions the line segment $\overleftrightarrow{AB}$ into two*

segments 

or two disjoint sets.

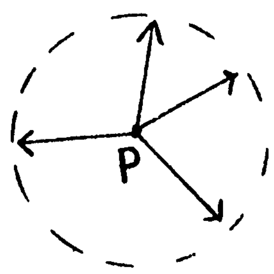


The line partitions the points of the plane into two half planes, H_1 *and* H_2.

5520 . . . . . . 2. **Locus of points**

Locus of points is any system of points which satisfies one or more conditions.

Ex. *The locus of points in a plane equidistant from a fixed point is a circle.*



Conditions:

Points lie in a plane.
There is a fixed point P.
All points in locus must be equidistant from P.

5992 . . Topic V: Measurement of Nongeometric Quantities

6000 . . . . A. Meaning of measurement (direct, indirect)

To measure means to compare an object with some suitable unit usually a standard unit.

Ex. Length is measured by a unit of length such as an inch or yard. Area is measured by a square unit such as a square foot.

6001 . . . . . . 1. Approximate nature of measurement

No measurement is exact. If you are measuring a line segment your measurement will be affected by the width of the dots at the endpoints, the angle at which you see the lines on your ruler, worn edges of your ruler and so on.

6003 . . . . . . 2. Precision

The smaller the unit of measure the greater the precision. *See 6005, p. 80.*

If the unit of measure is 1/2 inch and something is measured to the nearest 1/2 inch, the precision of the measurement is 1/2 inch.

6005 . . . . . . 3. The greatest possible error

In any measurement the greatest possible error is 1/2 the smallest division (unit) used on the measuring instrument.

Ex. The greatest possible error in measuring 5 inches with a ruler marked to half inches is 1/4 inch.

6007 . . . . B. Units of Measure

6008 . . . . . . 1. Historical development of units of measure

6009 . . . . a. Non-standard units such as foot, cubit, furlong leading to the standardized English system

6010 . . . . b. Metric units

6025 . . . . 2. Linear units of measure

6028 . . . . a. Non-standard

6030 . . . . b. English units for yards or less

6032 . . . . c. Metric units for meters or less

6034 . . . . 3. Square units of measure in the English system of measures

6035 . . . . 4. Square units of measure in the metric system of measures

6036 . . . . 5. Cubic units of measure in the English system

6037 . . . . 6. Cubic units of measure in the metric system

6038 . . . C. Need for modern units of measure

Ex. decibel, light years

6040 . . . D. Money

6050 . . . E. Time

6060 . . . F1. Distance in English units for lengths longer than a yard

6065 . . . F2. Distance in metric units for lengths longer than a meter

See 5260 for measurement of line segments.

6070 . . . G1. Liquids in English units

6075 . . . G2. Liquids in metric units

6080 . . . H. Temperature: Fahrenheit and centigrade

6090 . . . I1. Weight in English units

6095 . . . I2. Weight in metric units

6100 . . . J. Dry measures

6110 . . . K. Quantity (dozen, gross, etc.)

6120 . . . L. Operations related to denominate numbers

Ex.

$$\begin{array}{r} \text{3 hr. 10 min.} \\ + \underline{\text{2 hr. 50 min.}} \\ \text{6 hr.} \end{array}$$

Code 6120 and 6050.

$$\begin{array}{r} \text{6 lb. 10 oz.} \\ - \underline{\text{4 lb. 15 oz.}} \\ \text{1 lb. 11 oz.} \end{array}$$

Code 6120 and 6090.

6130 . . . . M. Conversion to other standard units measuring several kinds of nongeometric quantities

Ex. In one lesson:

10 pecks $\stackrel{m}{=}$ $\underline{2}$ bushels $\underline{2}$ pecks

90 minutes $\stackrel{m}{=}$ $\underline{1}$ hour $\underline{30}$ minutes

15 quarts $\stackrel{m}{=}$ $\underline{3}$ gallons $\underline{3}$ quarts

etc.

Note: If conversion is being developed with one kind of nongeometric quantity only, code under the quantity.

Ex. 21 days $\stackrel{m}{=}$ $\underline{3}$ weeks

120 minutes $\stackrel{m}{=}$ $\underline{2}$ hours

24 months $\stackrel{m}{=}$ $\underline{2}$ years

3 days $\stackrel{m}{=}$ $\underline{72}$ hours

etc. *Code 6050.*

6992 . . Topic VI: Number Patterns and Relationships

6994 . . . . A. Elementary number theory

7000 . . . . 1. Odd and even numbers

Even numbers are the integers divisible by 2.

$E = \{\cdots, {}^{-}4, {}^{-}2, 0, {}^{+}2, {}^{+}4, \cdots\}$

Odd numbers are integers not in the set of even numbers.

$0 = \{\cdots, {}^{-}3, {}^{-}1, {}^{+}1, {}^{+}3, {}^{+}5, \cdots\}$

7010 . . . . 2. Factors and primes

In any statement such as 5 x 8 = 40, 5 and 8 are called ***factors****.*

A prime number is a whole number that has only two integral factors, itself and 1.

Ex. 2,3,5,7,11,13,···. The number 1 is usually excluded as a prime since it has only one factor.

7020 . . . . 3. General composite numbers

A composite number is a whole number which has more than the two factors, itself and 1. A natural number greater than 1 which is not a prime number is a composite number.

Ex. 12 is a composite number since its factors are 1, 2, 3, 4, 6 and 12.

7030 . . . . 4. Special composite numbers

A perfect number is an integer which is equal to the sum of all of its factors excluding itself.

Ex. $6 = 1 + 2 + 3$; $28 = 1 + 2 + 4 + 7 + 14$

Relatively prime numbers have no factor except unity in common.

Ex. $8 = 1 \times 2 \times 2 \times 2$

$9 = 1 \times 3 \times 3$

Both 8 and 9 are composite numbers but relatively prime to each other.

(contd)

7030 (contd) *Numbers are amicable numbers if the sum of the factors of each number (excluding itself) equals the other number.*

Ex. *220 and 284*

The factors of 220 are 1,2,4,5,10,11, 20,22,44,55,110; their sum is 284.

The factors of 284 are 1,2,4,71,142; their sum is 220.

7050 . . . . 5. Greatest common factor

The greatest number which is a factor of each of two or more natural numbers is their greatest common factor.

Ex. *4 is the GCF of 8 and 12.*

7055 . . . . 6. Multiples

Multiples of a number N are numbers (products) obtained by multiplying N by integers.

Ex. *10, 35, 125, and 5000 are multiples of 5.*

7060 . . . . 7. Least common multiple

The least common multiple (also lowest common denominator, LCD, of fractions) of two or more natural numbers is the least natural number exactly divisible by all of the numbers.

Ex. *The LCM of 4, 10, and 12 is 60.*

7070 . . . . 8. Unique factorization (prime factorization)

Unique factorization or complete factorization occurs when the number is expressed as the product of its prime factors.

Ex. *3 x 4 = 12 shows 3 and 4 as factors of 12 but the expression 3 x 2 x 2 = 12 shows complete factorization.*

7080 . . . . 9. Rules for divisibility

All even numbers can be divided exactly by 2.

All numbers represented by numerals ending in 0 or 5 are exactly divisible by 5.

All numbers represented by numerals ending in 0 are exactly divisible by 10.

If the sum of the numbers named by the digits in a base 10 numeral is exactly divisible by 3 then the number is divisible by 3.

Ex. *288 is divisible by 3 since the sum 2 + 8 + 8 or 18 is divisible by 3.*

Proof: $2x(100) + 8x(10) + 8 =$

$2x(99+1) + 8(9+1) + 8 =$

$2x99 + 2 + 8x9 + 8 + 8 =$

$(2x99) + (8x9) + 2 + 8 + 8 = 288$

The first two terms are divisible by 3 so the number is divisible by 3 if (2 + 8 + 8) is divisible by 3.

Rules for divisibility by 4,6,8, and 9 are often used, also.

Reference: *Peterson & Hashisaki:* *Theory of Arithmetic* *- John Wiley & Sons, copyright 1963, pages 126-127.*

7082 . . . B. General number sequences and patterns

Number sequences are numbers given in some order, usually according to a pattern.

Ex. $1,1,2,3,3,4,5,5,6,\cdots$ $\quad 3,2,4,3,5,4,6,5,\cdots$

$0,3,8,15,24,\cdots$ $\quad 1,3,4,7,11,18,29,\cdots$

7088 . . . C. Special number sequences

7090 . . . . 1. Arithmetic progressions

An arithmetic progression is a sequence of numbers each differing from the preceding number by a fixed amount.

(contd)

7090 (contd) *Ex.* *3,6,9,12,··· the constant difference is 3;*

8,6,4,2,··· the constant difference is $^{-}2$.

Use code 7090 when arithmetic progressions are so called by the authors. For skip counting in primary grades use code 0080.

7100 . . . . 2. Geometric progressions

A geometric progression is a sequence of numbers each of which differs from the preceding number by a constant factor.

Ex. *1,3,9,27,···the constant factor is 3.*

32,16,8,4,2,1,1/2,1/4,···the constant factor is 1/2.

7110 . . . . 3. Triangular numbers

A triangular number is the cardinal number of a set of dots used in making triangular arrays beginning with one dot and continuing with rows of 2,3,4,··· dots.

Ex. *Using the first two rows of the diagram 3 is seen to be a triangular number. Using the first three rows 6 is seen to be such a number.*

```
   .
  . .
 . . .
. . . .
```

7120 . . . . 4. Square numbers

Square numbers are the cardinal numbers of square arrays

Ex.

```
       . . .
. .    . . .
. .    . . .
```

4 and 9 are such square numbers.

7130 . . . . 5. Factorial numbers

Factorial numbers are numbers symbolized by $n!$ *or* $\lfloor\underline{n}$ *to indicate the product of a series of consecutive positive integers from 1 to the given number.*

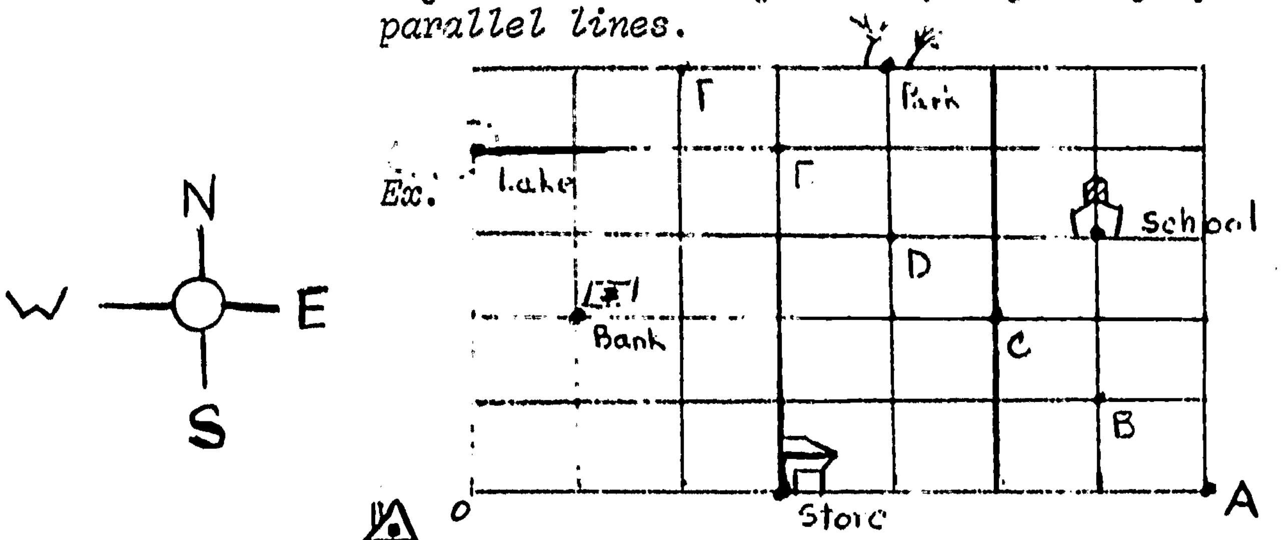
Ex. $3! = 1 \times 2 \times 3 = 6$ *and is read*

Factorial $3 = 1 \times 2 \times 3 = 6$.

7140 . . . . 6. Grid

A grid is an arrangement of regularly spaced parallel lines.

Ex.



How many blocks east must John go to get to point A?

How many blocks east and how many blocks north does he go to point C?

Note: Magic squares and practice exercises for basic facts in a grid arrangement are coded under the operation involved. The behavior code will indicate whether the practice uses magic squares or similar grid ideas.

7150 . . . . 7. Fibonacci numbers

Fibonacci numbers are numbers in the sequence 0,1,1,2,3,5,8,$\cdots$. Each number beginning with the third is secured by finding the sum of the two preceding numbers. Leonardo Fibonacci was a mathematician of the 13th Century who wrote treatises on the theory of numbers. His name was attached to the above series.

7160 . . . D. Special patterns (including short cuts)

Code with an operation if possible.

Ex. *To multiply by 25 quickly, multiply by 100 and divide by 4 (actually multiplying by* $\frac{100}{4}$*, another name for 25).*

Ex. $45^2 = (40+5) \times (40+5)$
$= (40 \times 40) + (10 \times 40) + (5 \times 5)$
$= (50 \times 40) + 25 = 2025$

Using the short cut

$45^2 = (5 \times 4) \times 100 + 25$ *or* 2025

Code 7160 and 0700 (raising to powers and finding roots).

(contd)

7160 (contd)

Ex. $15 \times 15 = 225$
$35 \times 35 = 1225$
$65 \times 65 = 4225$
$75 \times 75 = 5625$
$95 \times 95 = 9025$
$45 \times 45 = 2?25$
$85 \times 85 = ????$

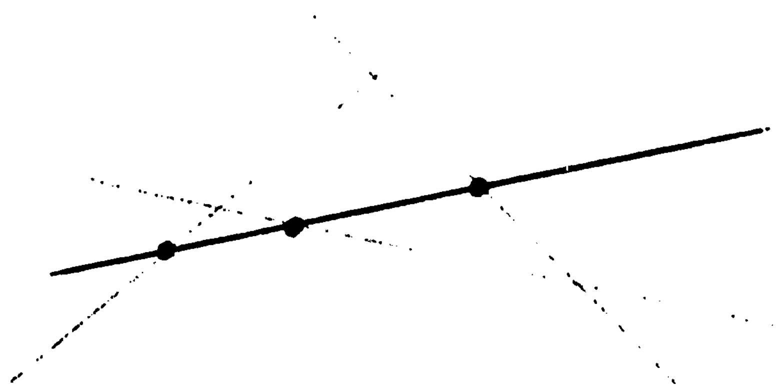
There is an easy way to find the product when a number is multiplied by itself if the numeral for the number has a 5 in the ones place. Can you see the pattern?

Let t = tens digit
Let 5 = ones digit

$(t + 1) \times t \times 100 + (5 \times 5) = N$

Ex. *Since* $3 + 7 = 10$, *then* $13 + 7 = ?$
Since $8 + 7 = 15$, *then* $18 + 7 = ?$
Since $8 + 9 = 17$, *then* $18 + 9 = ?$
Since $9 + 6 = 15$, *then* $19 + 6 = ?$
Since $4 + 9 = 13$, *then* $14 + 9 = ?$

Ex.



In this figure there are ? lines. The heavy line intersects each of the other lines in ? point(s).

Does each line intersect every other line in the same number of points?

To find the greatest number of points of intersection determined by 4 lines, multiply the number of lines, ?, by the number of points of intersection on each line, ?, and then divide by ?. There are $\frac{? \times ?}{2}$, *or* $\frac{?}{}$ *points.*

Try this with 5 lines, 6 lines, 3 lines, n lines.

7992 . . Topic VII: Other Topics

8000 . . . . A. Ratio and proportion

A ratio is a comparison of two numbers by division.

A ratio is a fractional number used to compare the cardinal numbers of two disjoint sets.

Ex. *The ratio of set A to set B is* $\frac{2}{5}$.

$A = \{● ●\}$

$B = \{○○○○ ○\}$

A ratio is a comparison between two quantities which have the same dimensions, expressed in the same unit.

Ex. *Larry has 4 books and John has 7 books. The ratio of Larry's books to John's is 4 to 7 or* $\frac{4}{7}$.

A ratio of 1 to 2 can be represented by any members of the set $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \cdots\right\}$. *It may be expressed 1 to 2,* $\frac{1}{2}$, *1:2.*

A statement which shows that two ratios are equal is called a ***proportion****.*

Ex. $\frac{2}{3} = \frac{4}{6}$ $\qquad\qquad$ $\frac{2}{3} = \frac{x}{18}$

8002 . . . . 1. Rate pairs

A rate is a comparison between two quantities having different dimensions such as miles per hour.

Ex. *If one candy bar costs 6¢, 2 candy bars cost ?¢.*

Note: *Most verbal problems using multiplication involve the concept of rate.*

See 4035, p. 62.

8003 . . . . B. Per cent

8004 . . . . 1. Meaning and vocabulary

8005 . . . . 2. Developed through use of ratios

8006 . . . . 3. Developed through use of equations

8007 . . . . 4. Developed through use of the formula P = b x r (percentage equals base times rate)

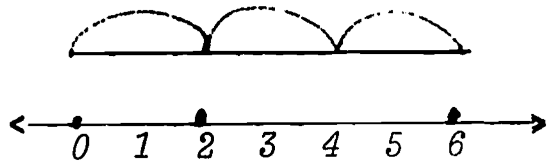
8008 . . . . 5. Computation related to per cent

8012 . . . . C. Graphs

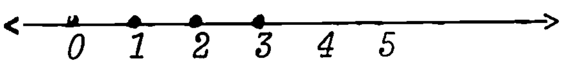
8020 . . . . 1. Solution sets of equalities and inequalities on the number line

Ex. $3 \times \square = 6$ *The solution set is {2}.*

The number line shows



Graph the inequality $3 \times \square < 10$ if the Universal set is the set of whole numbers. The solution set by dots is {0,1,2,3}. The number line graph is shown.



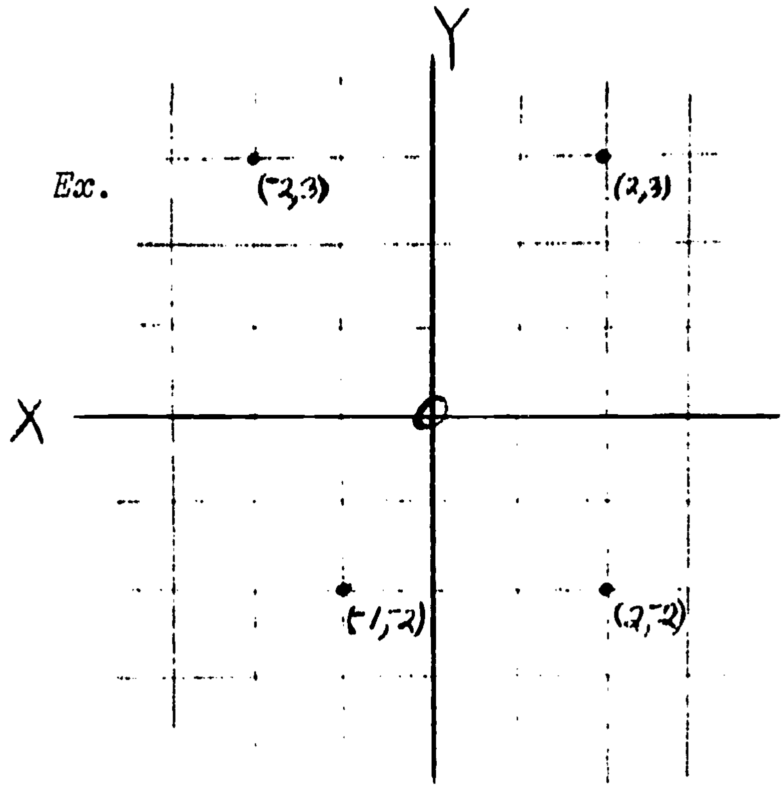
8030 . . . . 2. Ordered pairs on a coordinate plane

Over 300 years ago Descartes envisioned a plane (surface) on which pairs of numbers were used to locate points. This plane is called the Cartesian or coordinate plane. Ordinary graph paper illustrates such a plane. The pairs of points are ordered so that the first number represents the horizontal or X distance and the second number represents the vertical or Y distance.

Ex. *The X and Y axes drawn on the plane may be considered as two number lines with the 0 point at their intersection. The ordered pair (2,3) locates a point 2 units to the left of the 0 point and 3 units above it.*

(contd)

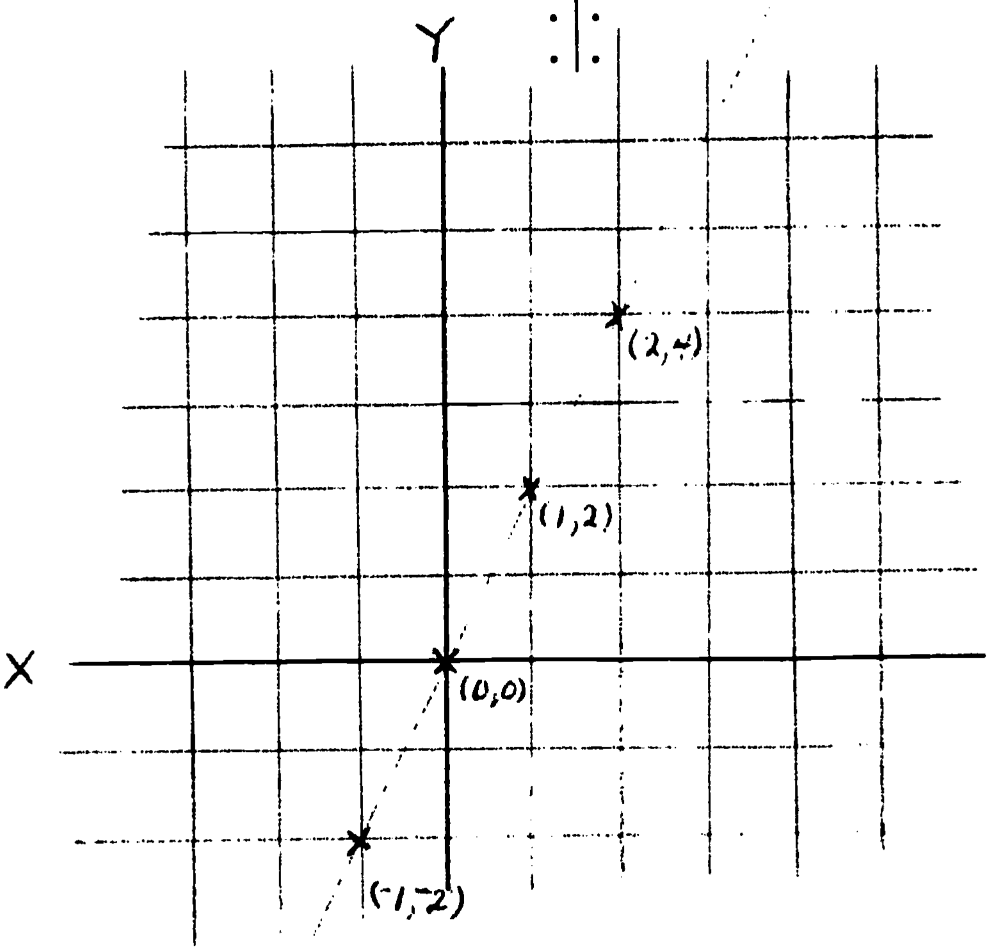
8030 (contd) *Ex.*



8040 . . . . . . 3. Solution sets of equalities and inequalities on a coordinate plane

Equality $\{(x,y)|y=2x\}$

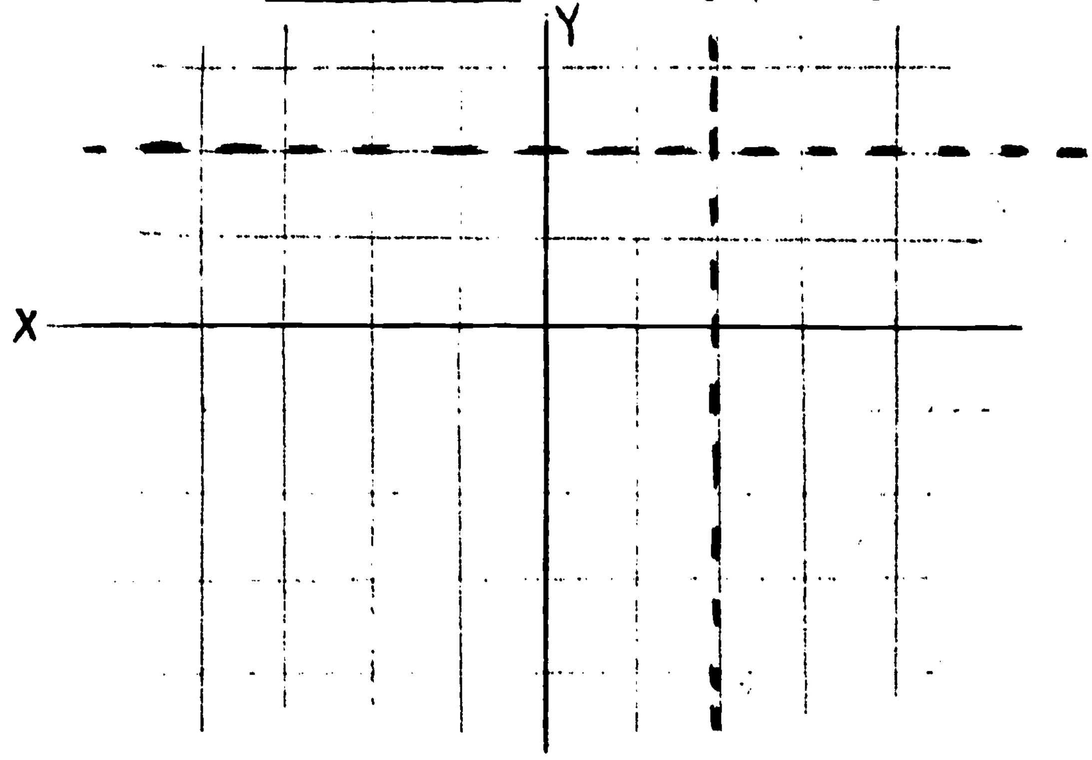
x	y
-1	-2
0	0
1	2
2	4
3	6
.	.
.	.
.	.



Find ordered pairs like those given. Plot them on the coordinate plane. Connect the points. A straight line will result.

(contd)

8040 (contd) *Inequalities* $\{(x,y)\,|\,x>2,\ y<2\}$



Since x=2 is not part of the graph that line is dotted. In like manner y=2 is dotted. Where the fine lines cross we have an area extended indefinitely to the right and down when the graph is greater than 2 for x and less than 2 for y.

8044 . . . . D. Descriptive statistics

8050 . . . . 1. Frequency tables, charts, graphs (bar, line, circle, dot, picture, etc.)

8060 . . . . 2. Measures of central tendency: average, mean, mode, median

The arithmetic <u>mean</u> of a sequence of numbers is an average and is found by dividing the sum of the numbers by the number of items in the sequence.

Ex. The A.M. of 4,8,10, and 16 is $(4+8+10+16) \div 4$.

The <u>mode</u> of a sequence of numbers is the number or category that occurs most often.

The <u>median</u> of a sequence of numbers is the middle score in the sequence after the scores have been arranged from lowest to highest or highest to lowest. The median of the scores 5,6,8,12,18,20, and 24 is 12.

The median of scores 4,5,6,8,11,12,18,24 is assumed to be 1/2 the sum of the two middle terms 8 and 11.
$\frac{1}{2}(8+11) = 9\frac{1}{2}$.

8070 . . . . 3. Measures of variability: range, quartiles, percentiles, average deviation, standard deviation

The range of a sequence of numbers is the interval between the least and the greatest of a set of quantities.

Ex. The range of the series 1,3,7,10,15 is 15 -1 or 14.

The first quartile Q_1 is the point below which lie 25% of the scores.

The third quartile Q_3 is the point below which lie 75% of the scores.

The 20th percentile is the point below which lie 20% of the scores.

The 50th percentile is the point below which lie 50% of the scores.

The average deviation is the arithmetic mean of the deviations of all the separate measures from the arithmetic mean. It is found by using the formula $A.D. = \frac{\Sigma|x|}{N}$.

The absolute value of the sum of the deviations divided by the number of deviations is the average or mean deviation.

The standard deviation for ungrouped data is found by using the formula $\sigma = \sqrt{\frac{\Sigma x^2}{N}}$ *where x is the deviation of each score from the mean, Σx^2 is the sum of the deviations squared, N is the number of terms.*

8080 . . . . E. Probability

If several events are equally likely to happen, the chance (probability) that a given event will happen is the ratio of the favorable possibilities to the total possibilities. The probability that a 3 will show on one toss of a die is 1/6.

Only one 3 can appear. Any of six numerals may appear.

8084 . . . . F. **Finite mathematical systems**

Mathematical systems having bounds or ends.

Systems which may be completed by counting.

Ex. The system of numbers on the clock is a finite system.

A system using only 5 numerals is a finite system.

8100 . . . . 1. **Modular arithmetic (clock arithmetic)**

Modular arithmetic is based upon a set of finite numbers.

Ex. The usual clockface has only 12 numbers. Hence, in that number system we may write $8 + 5 = 1$. A movement of the hand 5 spaces beyond 8 brings the hand to 1.

Note: Use of the clock to teach the base 10 system of numeration is coded 3050. Telling time is coded 6050. Use this code for finite mathematical systems only.

8110 . . . . 2. **Without numbers**

Letters or geometric figures may be used.

8120 . . . . 3. **Other**

8130 . . . . G. **Logic**

Logic may be described roughly as the study of necessary inferences or compelling conclusions.

The mathematical study of correct methods of deductive reasoning is the study of logic.

Ex. If it has rained the ground is wet.

It has rained.

∴ the ground is wet.

If $a \times b = n$ *and* $a = b$ *then* $b \times b = n$ $b^2 = n$ *and*

$a \times a = n$ $a^2 = n$.

8140 . . . . H. Functions and relations

A function is a correspondence that associates with each number X of a given set of numbers one and only one number Y.

A function is a set of ordered pairs of numbers such that for each first value there is one and only one second value.

Ex. *In the formula for the circumference of a circle, $C = 2\pi r$, for every value of r there is one and only one value for C. C is therefore a function of r.*

Ex.

a	$^-3$	7	10	23	31	46
b	$^-5$	15	21			

What is the pattern used to find b? $b = 2a + 1$.

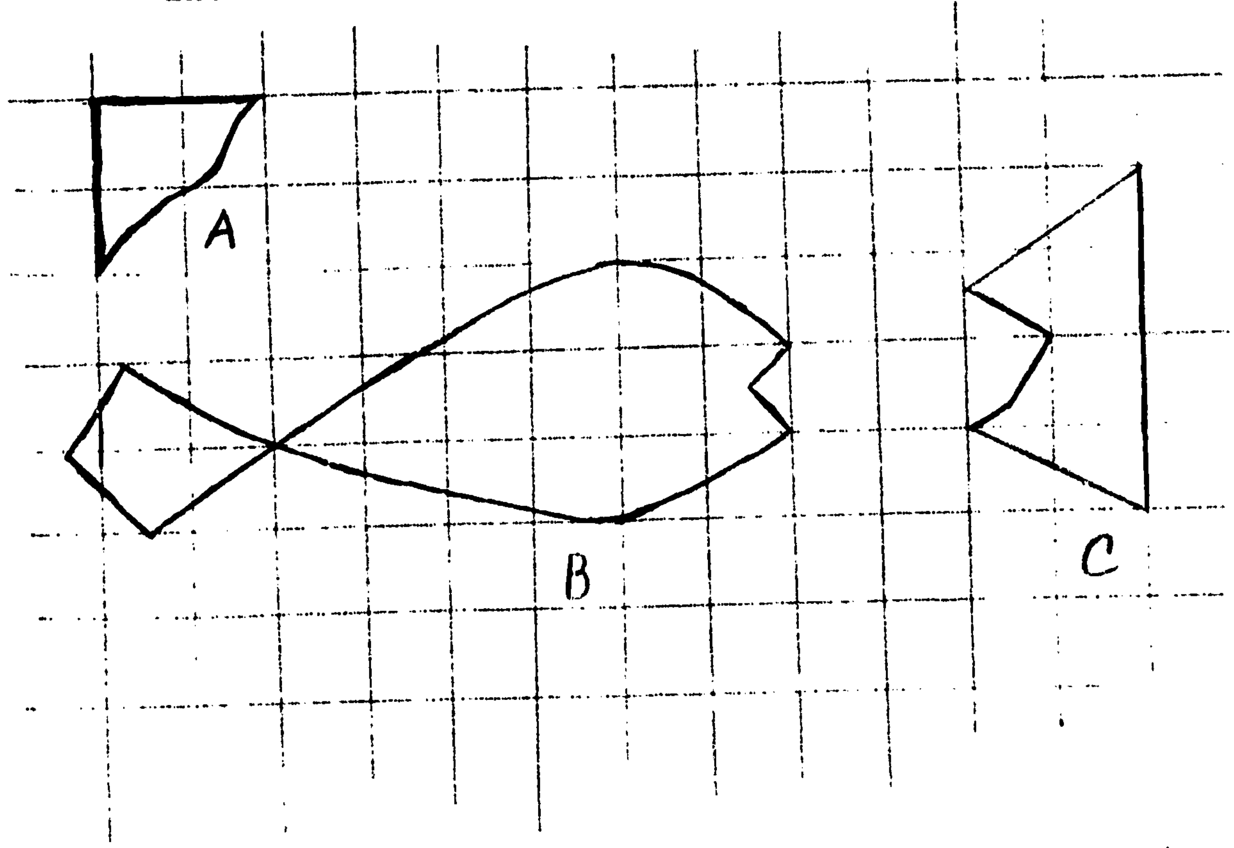
8150 . . . . I. Estimation

An approximate (estimated) answer to a problem can often be found by using round numbers and mental computation.

Ex. *The sum of* 428 *is approximately* 400 + 400 + 200 = 1000

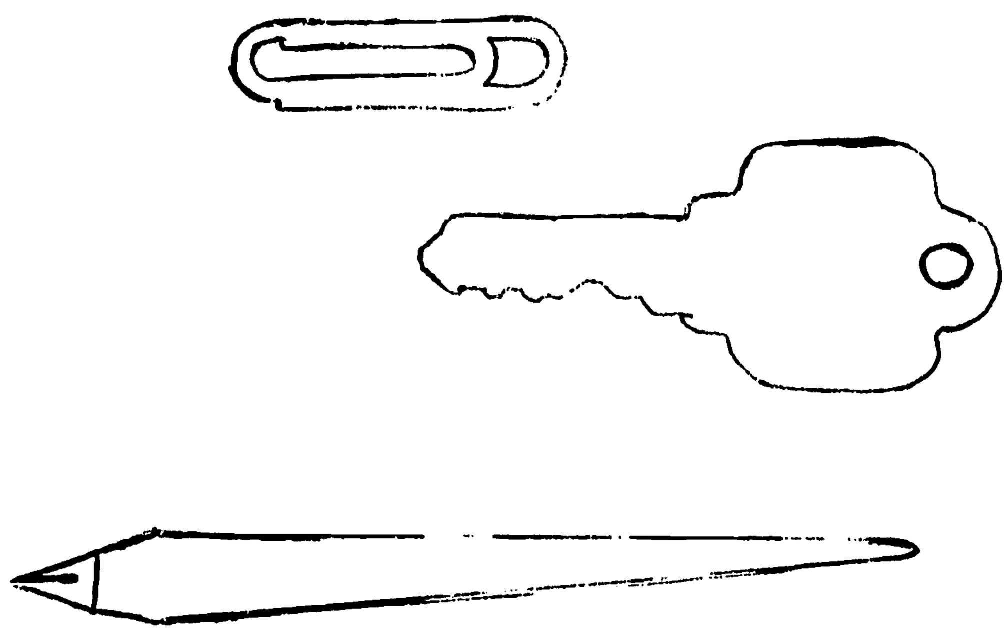
$$\begin{array}{r} 428 \\ 365 \\ +215 \\ \hline 1008 \end{array} \leftarrow \text{actual sum}$$

Ex. *Estimate the area of each region*



(contd)

8150 (contd) *Ex: Estimate the length of each object to the nearest half inch.*



8160 . . . . J. **Relations (properties of)**

A relation is a set of ordered pairs of numbers such that for each first value there may be one or more than one second value. (4,2) and (4,$^-$2) both satisfy this relation.

$$\{(x,y) \mid y^2 = x\}$$

Some relations in the set of natural numbers are: is greater than, divides, is a factor of, is relatively prime to, is a subset of, is equivalent to, is equal to.

Some examples in geometry are: is parallel to, is congruent to, is perpendicular to, is symmetric to.

8170 . . . . K. **Mathematical sentences (equations)**

An arrangement of symbols indicating that a relation exists between two or more things. The sentence contains at least two symbols for numbers, points, sets, or the like and a relation symbol. The most common relation symbols are =, > and <.

(contd)

8170 (contd) *Ex.*

	symbol for thing	relation symbol	symbol for thing	
Equations:	$2 + x$	$=$	3	
	$3y$	$=$	18	
Inequalities:	2	$>$	1	
	$\frac{x}{y}$	$<$	7	
	$3 + 5$	$\neq$	10	
Other:	$\overleftrightarrow{AB}$	$\perp$	$\overleftrightarrow{CD}$	*line AB is perpendicular to line CD.*
	$\{1,2\}$	$\subset$	$\{1,2,3,4\}$	*$\{1,2\}$ is a subset of $\{1,2,3,4\}$.*

Kinds of Sentences:	*Open Sentence*	*Statement*
	$x + y = 13$	$2 + 11 = 13$
	$4 \square 9 = 36$	$4 \times 9 = 36$
	$6 \times 5 \bigcirc 24$	$6 \times 5 > 24$

Developmental work with problem-solving may be classified under code 8170. Problem-solving (application) should be coded under the operation involved.

8992 . . Topic VIII: Summaries

9000 . . . . A. General review

Work previously presented and now being reinforced is considered as review. Some time has lapsed between developmental work and this lesson. It usually appears at end of chapter or in a following chapter.

Work immediately or very shortly following a lesson to further strengthen and develop concepts just presented is considered practice.

Use code 9000 if more than two mathematical content topics are presented in a single review lesson and one or two items of content cannot be identified as of major importance.

Sets of supplementary pages at the end of a book are coded 9000 for general review or 9020 through 9111 for specific review.

9010 . . . . B. Test

9020 . . . . C. Review properties of and basic operations with whole numbers

9030 . . . . D. Review properties of and basic operations with fractional numbers

9040 . . . . E. Review properties of and basic operations with integers

9050 . . . . F. Review of numeration

9060 . . . . G. Review of sets

9070 . . . . H. Review of geometry

9080 . . . . I. Review of measurement

9090 . . . . J. Review of number patterns and relationships

9100 . . . . K. Review of other topics

9101 . . . . 1. Ratio and proportion

9102 . . . . 2. Percent

9103 . . . . 3. Graphs

9104 . . . . 4. Statistics

9105 . . . . 5. Probability

9106 . . . . 6. Finite mathematical systems

9107 . . . . 7. Logic

9108 . . . . 8. Functions and relations

9109 . . . . 9. Estimation

9110 . . . . 10. Relations, properties

9111 . . . . 11. Mathematical sentences (equations)

Use when specifically being reviewed.

Mathematical sentences (equations) will be used with many topics and especially with problem solving. The lessons will then be coded according to the topic being studied.

9800 . . . . L. Appendixes

9810 . . . . 1. Bibliography - student

9820 . . . . 2. Bibliography - teacher

9830 . . . . 3. Games

9840 . . . . 4. Glossary

9850 . . . . 5. Index - student text

9860 . . . . 6. Index - teacher text

9870 . . . . 7. Introduction - foreward
preface
notes to teacher
philosophy and series development
information about authors

9880 . . . . 8. Mathematics text for teacher
inservice material

9890 . . . . 9. Overview of Program - Grade Level (survey)

9900 . . . . 10. Overview of Program - K-6
Scope and Sequence Chart

9910 . . . . 11. Table of Contents

9920 . . . . 12. Tables - Measures, etc.

9930 . . . . 13. Title Page, Covers, Notes, General

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